Radiative Processes 24/10/03

The simplest process we can imagine is scattering by a single, free electron for which the equation of motion is:

$$m_e \ddot{\underset{\sim}{x}} = -e\underset{\sim}{E} \rightarrow -\omega^2 m_e \underset{\sim}{x}_\omega = -e\underset{\sim}{E}_\omega$$

using $E = \underset{\sim}{E}_\omega e^{i\omega t}$ and similarly for x_ω. Then $e\underset{\sim}{x}_\omega = \underset{\sim}{d}_\omega$ is the dipole moment and from the Larmor equation,

$$\left(\begin{matrix}\text{emitted}\\ \text{power}\end{matrix}\right) \rightarrow P = \frac{2}{3}\frac{e}{c}|\ddot{\underset{\sim}{x}}|^2 \sim |d_\omega|^2\omega^4$$

for the power radiated by a dipole, we can define the cross section as

$$\sigma \sim \frac{P}{I_\omega}, \quad I_\omega \sim |E_\omega|^2$$

so you see that the cross section is <u>independent of frequency</u> (!) and of order $\frac{e^2}{mc^2}$; and since this is strictly classical $\hbar$ doesn't appear. Actually, notice that as soon as you know that $P \sim |\ddot{\underset{\sim}{x}}|^2$ the essential result follows immediately. The precise value is:

$$\sigma_T = 0.4\ \mathrm{cm}^{-2}\mathrm{g}^{-1} \quad \text{(Thomson cross section)}$$

as a scaling for ordinary matter (not degenerate). Why? The inertia is proportional to m_e, which is constant if $\hbar\omega \ll m_e c^2$, so the electron responds without delay to E_ω at all frequencies - there is no resonance frequency for an unbound particle. So here you have the <u>minimum</u> cross section for ordinary matter, and for a purely scattering medium, $\kappa_R = \sigma_T$ and τ_R is frequency independent.

It also follows that σ_T is isotropic; again, for a free electron, the radiated solid angle is 4π since the dipole moment is always induced.

One last point on line formation and transfer : Broadening

Any process that decreases the lifetime of an energy level is a broadening mechanism. This will enter directly into Γ, not as a simple convolution. There are, first, quantum effects that change line structure, the astrophysically most important is the Zeeman effect. Put simply, for a perturbation:

$$\Delta E = \langle \underset{\sim}{\mu} \cdot \hat{b} \rangle B$$

for a magnetic dipole moment $\underset{\sim}{\mu}$, a field $\underset{\sim}{B} = B\hat{b}$ produces a separation of the energy levels into spin states, depending on the coupling that forms the level. For hydrogen, you know this is especially simple - a triplet of lines for which $\Delta m_j = 0, \pm 1$ for which the separation ΔE is linear in field strength B. Since the precise form depends on $g(nlsj)$, the Landé g factor, and thus on the level classification (coupling scheme) I won't write this down precisely but all lines are potentially affected. For the typical fields observed in stars, $B \lesssim 10\,G$, this isn't important but for some cases it is. For example, for sunspots $B \sim 1\,kG$ and this is how the cosmic Zeeman effect was first observed (Hale, ca. 1900). A small fraction of stars in the $2 \lesssim M/M_\odot \lesssim 10$ range - all main sequence - show $B \sim 1-10\,kG$ (the largest is $\sim 30\,kG$, HD 215441) for which the splitting is large and global.

NB Since for a star you must average over the surface, the field observed is weighted by the surface area, Ω_B, subtended by the strong field region $(\Omega_B/2\pi)$ so for spots like those on the Sun, the net field is very small, $\sim 10^{-4} B_{max}$. For the strongly magnetic stars, called Ap and Bp [p = peculiar], $\Omega_B/2\pi \sim 1$ but these appear to be global, dipolar fields. Such stars are quite rare, $< 10\%$ of those in this mass range, and only one, θ^1 Ori C, is an O star. The origin of the fields is still a major puzzle since they are found even in cluster stars with ages $< 10^6$ yr.