Stellar winds : radiative and general

To begin with our first assumptions from today's lecture, we'll consider only spherical (1D) time independent (steady state) flows:

$$\frac{\partial v}{\partial t} = \frac{\partial \rho}{\partial t} = 0$$

so the equations are :

$$\frac{1}{r^2}\frac{\partial}{\partial r}\rho v r^2 = 0$$

$$v\frac{\partial v}{\partial r} = -\frac{1}{\rho}\frac{\partial P}{\partial r} + a_{net}$$

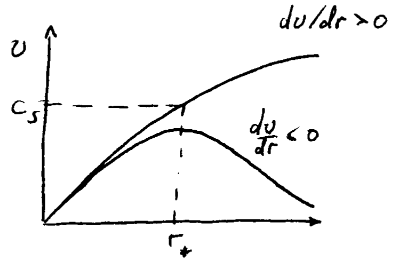
where as I said today, a_{net} is any external acceleration. Let's also allow P to be a generalized pressure, $P = \rho c_s^2$, but for this time I'll allow c_s to vary with distance r from the center. From the continuity equation

$$\rho v r^2 = J = \text{constant} \; ; \; \ln\rho + \ln v + 2\ln r = \text{const.}$$

which serves to eliminate ρ (or v) in terms of the other variable. Now

$$\frac{1}{\rho}\frac{\partial P}{\partial r} = \frac{c_s^2}{\rho}\frac{\partial \rho}{\partial r} + \frac{\partial c_s^2}{\partial r},$$

$$= c_s^2\left[-\frac{2}{r} - \frac{1}{v}\frac{\partial v}{\partial r}\right] + \frac{\partial c_s^2}{\partial r};$$



and it follows that :

$$\underbrace{\frac{1}{v}(v^2 - c_s^2)\frac{dv}{dr}}_{\mathcal{L}\frac{dv}{dr}} = \underbrace{+\frac{2c_s^2}{r} - \frac{dc_s^2}{dr} + a_{net}}_{\mathcal{R}}$$

Notice that $v \geq 0$ by definition and $\rho \geq 0$. Also, $\frac{2c_s^2}{r} \geq 0$. So as I said in class, if we require that $\mathcal{R}$ can change sign (from < 0, for instance, to > 0) and that dv/dr is everywhere smooth, then $\mathcal{L}\frac{dv}{dr} < 0$ when $\mathcal{R} < 0$ and $\mathcal{L}\frac{dv}{dr} > 0$ when $\mathcal{R} > 0$. For an isothermal region, one for which $t_{cool} < t_{flow}$, then $dc_s/dr \to 0$. Let's do that for now. Then

$$\frac{2c_s^2}{r} + a_{net} = 0 \to v = c_s \text{, the sonic point.}$$

Thus, if $a_{net} = -\frac{GM}{r^2}$, we find a critical r such that :

$$r_* = \frac{2c_s^2}{GM}, \quad \dot{M} = 4\pi r_*^2 \rho(r_*) c_s$$

and as M increases, r_* decreases so $\rho(r_*)$ increases. If $r_* \gg R$, $\dot{M}$ is negligible, but as r_* decreases, that's no longer true. And notice, in light of our discussion today, what happens if

$$a_{net} = g_{rad} - g.$$

Then when $g_{rad} < g$, $\frac{2c_s^2}{r} + a_{net}$ may be > 0 so $v > c_s$ even if radiation pressure doesn't balance gravity locally! Since

$$g_{rad} \sim \frac{1}{c}\int_0^\infty \kappa_\nu F_\nu \, d\nu$$

depends on abundance as well as (ρ, T), objects with the same thermodynamic properties may still have very different g_{rad}. Nowhere have we excluded the possibility that dv/dr can change sign, but if it does we have a problem for steady flows:

$$\frac{1}{v}\frac{dv}{dr} = -\frac{1}{\rho}\frac{d\rho}{dr} - \frac{2}{r} < 0 \Rightarrow \frac{d\rho}{dr} > 0$$

which means we build up a shell at large (?) distance of increasing pressure. This, in turn, must feed back on the flow since $v < c_s$ and a pressure wave can now propogate backwards, into the flow, and eventually the peak causes a static subsonic region to develop. If, on the other hand, when we're at $r > r_*$, a heat source is turned on, the flow may again accelerate.

If we're using a 1D plane parallel picture, in which we're looking in effect only at the flow in the region immediately around r_*, then:

$$(v^2 - c_s^2)\frac{1}{v}\frac{dv}{dx} = g_{rad} - g \rightarrow v = c_s \text{ at some } \tau_*$$

and therefore, as in class,

$$\dot{M} = 4\pi R^2 \rho(\tau_*)\, c_s(\tau_*)$$

In this case, we simply require the <u>local</u> gravitational acceleration, g, and <u>local</u> g_{rad}. A body may have $L < L_{Edd}$, which is a <u>global</u> quantity for minimal opacity, and still drive a wind since at some depth τ_* and lower (larger radius) $g_{rad} > g$ may apply. Such examples are frequently seen in luminous stars: on the main sequence, all stars with $M \gtrsim 10 M_\odot$ have outflows at the $10^{-8} M_\odot\, yr^{-1}$ level.
For a galaxy, as I mentioned in class:

$$g = -\frac{GM(r)}{r^2} = -\frac{d\Phi(r)}{dr}$$

so you need a model for the gravitational potential, $\Phi(r)$, or a mass model.
(A dark matter halo, for which $M(r) \sim r$, is especially simple since now for the Eddington case, $g_{rad} \sim \frac{1}{r^2}$ and $g \sim \frac{1}{r}$ so.

$$a_{net} = 0 \rightarrow \frac{GM_0}{r} = \frac{L}{4\pi r^2 \sigma_T c} \rightarrow \frac{4\pi G M_0}{\sigma_T L c} = r_*$$

or $\frac{L_{Edd}}{L} < 1$ may still drive an outflow).

So what happens to this wind? Taking v_∞ to be its terminal velocity, then L_W is constant and you have $R_s(t) \sim t^{3/5}$, the solution I discussed in class.