Esercizio 15/10/03

Since we have a path method for treating the radiative transfer, here's an example of how it can be used. Called the Monte Carlo method, it uses the probability notion for all processes and begins with this basic premise:

You can create a set, $\{P_i\}$, of random (well, really pseudo-random) numbers that allow you, at each step, to compute a path.

Assume you've generated some P_i. Then assume $\kappa_\nu + \sigma_\nu = 1$ (normalized coefficients) and $\tau = \tau_{scatt} + \tau_{abs}$. Then:

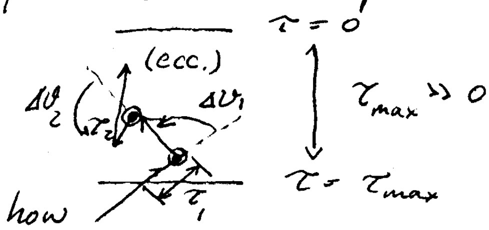
$$P_i = \exp - \left(\frac{\tau_i}{\mu_i}\right) p(\mu_i | \mu_j)$$

where now I'm writing the phase function, p, as a conditional probability to make clearer that only $\cos(\vartheta_i - \vartheta_j)$ matters. Then if μ is independent of τ, for each step i we generate a second random number, $P_i' = p(\mu_i | \mu_j)$. If p is uniform (i.e. constant) then $P_i' = p(\mu_i | \mu_j) \rightarrow P_i' = |\mu_i|$ independent of μ_j. But, as we'll see, p may depend on $\vartheta_i - \vartheta_j$. Take, for example,

$$p = a \cos^2(\vartheta_i - \vartheta_j) + b$$

which is called a dipole (you'll soon see why). Then $\cos^{-1}\left(\frac{P_i' - b}{a}\right) = \vartheta_i - \vartheta_j$ and, noting that $P_i' \rightarrow P_i'' P_i$, where P_i'' gives $\mathrm{sgn}(\vartheta_i - \vartheta_j)$ (take, e.g., $\Delta\vartheta > 0$ if $P_i'' > 0.5$, $\Delta\vartheta < 0$ if $0.5 > P_i''$ and $\Delta\vartheta = 0$ if $P_i'' = 0$, or use $\Delta\vartheta < \pi$ or $\Delta\vartheta > \pi$ or $\Delta\vartheta = \pi$, you're pretty free to define this). Each step then requires three (3) random variables. Thus:

$$\tau_i = -\mu_i \ln \frac{P_i}{p(\mu_i | \mu_j)}$$



is then the step in optical depth. I hope you see how this will proceed. Notice how you rotate each path. That's for convenience, the process is amazingly simple. For instance, if $\kappa_\nu = 0$ and $\sigma_\nu = \sigma$ and $p = \mathrm{const.}$, then $\Delta\vartheta = 2\pi P_i'$ and $\mu_i = \cos 2\pi P_i'$. The reason this is more difficult for an absorbing atmosphere is the need to separately treat the re-emission process. For now, we'll ignore this.