There exist several fundamental distance calibrators for cosmology, but they all rest on something local — hence our emphasis on the Galaxy. The logic runs something like this, and for once I'll do it in the form of a diagram.

Parallaxes, proper motions and radial velocities of solar neighborhood main sequence stars

binaries

clusters (Hyades, Coma, $\alpha$ Per, Pleiades) also moving groups, OB assoc.

Giants and turn-off point

binaries

Classical Cepheids

RR Lyr stars

globular clusters

ages; limit on age of Galaxy and, thus, universe

oldest halo stars and nuclear cosmochronology

zero point calibration

LMC, SMC Period-luminosity relation

novae

Local group galaxies

supernova remn.

Planetary nebulae

HII regions

Hubble-Sandage vars.

Virgo cluster

SN Ia calibration

Tully-Fisher relation ($\Delta v_{HI} - \mathcal{L}$)

Faber-Jackson relation ($\sigma - \mathcal{L}$)

secondary calibrators

surface brightness

galaxy lumin. function

linear sizes

clusters of galaxies

If this looks complex as a start, it is only the beginning. I'm actually simplifying the diagram considerably leaving out many fundamental steps (some are discussed in Ch. 8 of Tapestry but not all). For example let's look at one point I didn't mention in class. Eclipsing binaries have the feature that $M$, $\mathcal{L}$, and $R$ can be measured directly. Hence it is possible to obtain absolute (bolometric) luminosities for such systems beyond the LMC; with the VLT ($(M-m) \simeq 24^m$ for M31) it is/should be possible to reach B stars in M31, NGC 205, and M32. It's already possible to do this for the LMC/SMC. You can also use this to obtain the $M - \mathcal{L}$ relation for stars of different metallicities and calibrate distances to Galactic clusters.

Assumptions (some of them anyway)

1. No serious changes in calibrators with lookback time (generally no strong dependence on metallicity)
2. No fundamental environmental changes
3. Population invariance - no strong mixing or interactions with time
4. Invariance of physical constants

How do we compute evolutionary changes? The simplest, wrong, way would be to simply take a rate $\psi$ of star formation and use

$$B(m,t) = \varphi(m)\psi(t)$$

as the birthrate (by assertion, this is separable) with $\varphi(m)$ being a constant (stationary) stochastic function. Then we could ~~sim~~ sample $\varphi(m)$ by a Monte Carlo procedure with a modulating rate $\psi$ and compute isochrones. If $\psi$ is constant, then the HR diagram simply fills up over time as more older (low mass) stars populate the giant branch. The high mass stars, for which the MS lifetimes $\tau_{MS}(m > m_*)$ are short will reach a steady state but the low mass MS stars will accumulate for $m \ll m_*$ for some $m_*$ such that $\tau_{MS}(m_*) \simeq \Delta t$, the time interval we're considering. This is actually independent of the method. There will, in this picture, always be stars at all masses in the galaxy. The extreme opposite occurs, for instance, for a cluster of stars for which the $\psi(t) \to \delta(t)$, an instantaneous burst. Then, at any time, $\tau_{MS}(m)$ is steadily increasing with $\Delta t$ and the stars with $m > m_*$ simply vanish* (die somehow). Depending on the $\varphi(m)$, the brightest stars dominate the visible and UV spectra while the red and IR are given by the lowest mass stars, of which there are many. Now as an example, at $0.1\,M_\odot$ (near the MS lower limit) the luminosity is $\simeq 10^{-6}$ (or precisely $10^{-5.5}$) $L_\odot$ but for a Salpeter mass function, there are only $10^{+2.4}$ of them. So the high mass stars win out. With $\psi$ = constant, in other words, because you always have the same upper mass end, there is apparently no evolution.

$\Rightarrow$ But this is obviously wrong. These stars don't disintegrate just at the right moment. And something else changes: the metallicity, $Z$, of the stars comes from previously active stellar nucleosynthetic engines. This isn't just poetry: $Z(t)$ governs the evolutionary changes even if $\psi$ = const.

Question: How does $Z(t)$ affect the mass scale ($M_J$), the rate of stellar evolution, and the effective colors and luminosities for $\varphi(m)$ stationary? This is a very general question but relates directly to the core issue of why you must include metallicity evolution when modeling evolutionary corrections.

Now we can begin altering ~~all~~ our picture. First, stars don't return all their mass to the ISM: some remains locked in remnants and this depends on mass. So

$$-\frac{dM_g}{dt} = (1-R)\psi$$

where:

$$R = \int \varphi(m) r(m)\, dm \Big/ \int \varphi(m)\, dm$$

is the fraction of stellar mass returned. We're going now to use the instantaneous recycling picture in which all stars evolve in a very short time (Tinsley 1974, ApJ). Though evidently wrong again, since the age of the Galaxy is about that of a $0.4\,M_\odot$ star, for now we can make $r(m \ll m_\odot) \simeq 0$ to account for this. Nowhere do we need $\tau_{MS}(m)$ if we scale $r$ in this way. We make one further assumption. Since $ZM_g$ is the mass of metals,

$$\frac{d}{dt} ZM_g = \frac{dM_g}{dt}\frac{d}{dM_g}(ZM_g) = -(1-R)\psi \frac{dZM_g}{dM_g} ;$$

said differently, $M_g$ and $t$ can be exchanged. This is the closed box model, and it's immediately evident that $Z$ is a monotonic function of time since obviously $M_g$ steadily decreases. Unless, that is, there's always destruction of the species in stars, in which case $Z \downarrow$ as $M_g \downarrow$.

NB: we'll see later more details for this but for now, the best example of this is $^2D$, a product only of cosmological nucleosynthesis with no known stellar sources. Then with no sources and only sinks, $^2D/H$ decreases on the timescale $\psi^{-1}$. An alternate is, say, $^{56}Fe$ – synthesized in SN Ia and SN II explosions, for which $(H, {}^4He) \to {}^{56}Fe$ with some yield $Y$ of the stars. Then:

$$\frac{d}{dt}(Zm_g) = -(1-R)\psi\left(m_g \frac{dZ}{dm_g} + Z\right) \qquad \mu = \ln m_g$$

$$\Rightarrow -(1-R)\psi \frac{dZ}{d\mu} = (1-R)\psi \frac{d(1-Z)}{d\mu} = YR(1-Z)\psi$$

since

$$\frac{d}{dt}(Zm_g) = -(1-R)\psi Z + RY(1-Z)\psi :$$

$$\ln(1-Z) = \frac{RY}{1-R}\mu$$

and since $R < 1$, $1-Z \to 0$ as $\mu \to -\infty$ ($m_g \to 0$); but since $Z \ll 1$, we generally find that

$$Z \sim -\frac{RY}{1-R}\ln[m_g/m_g(0)]$$

Taking $\psi \sim m_g^n$ then gives $m_g(t)$ so $\psi(t)$ steadily decreases as $t$ increases and $Z(t)$ steadily increases.

The last step, assuming a functional form for $\psi(t) \to \psi(m_g)$ is called the Schmidt (1957, 1962) law found by assuming

$$\psi \sim N_{OB}\langle m_{OB}\rangle / \langle \tau_{MS,OB}\rangle$$

in any region and correlating it with the amount of HI gas we see. Yet again, this is wrong: stars do not form directly from diffuse neutral hydrogen, but Schmidt found that

$$\psi \sim m_g^2$$

nicely approximated his data. Actually, the law now used is a variant of this,

$$\psi \sim \Sigma_g^{1.4} \qquad (\text{or rather } \Sigma_g^n \; 1<n<2).$$

The supposed connection with Q, the Toomre parameter, is then suggestive since the self-gravitation is due to $\Sigma_g$, not $\rho_g$; here "g" indicates all forms of gas – including molecular – rather than just HI.

NB But again we encounter a problem: if $\psi \sim m_g^n$ then the rate is varying approximately as $(1-kt)^{-2}$ with time. Thus there should be a vast number of very low Z stars, which is not seen. There are many plausible solutions, since this is a generic problem for all closed models, the direct one being to "open" the box with some inflow rate:

$$\frac{dm_g}{dt} = -(1-R)\psi + f$$

which also makes the metallicity evolve more slowly (Larson, Tinsley, Pagel, Clayton; all at various times, using f makes a reasonable dilution of the existing gas and reduces the consumption). Alternatives include a stage of pre-enrichment (also called Population III or pre-galactic star formation) and/or a secular change in the IMF ($\varphi(m) \to \varphi(m,t)$ while keeping $B = \varphi\psi$ as a separability).

Any of these gives a reasonable solution. Note also that the Pop III solution has substantial cosmological (evolutionary) consequences since this generation will have the lowest opacity and, therefore, the highest $L_{uv}/M$ ratio. The formation of much enhanced ionized regions must result. We'll see this again with the formation of structure in the early universe, the so-called "reionization" epoch.

Question: Consider a single SN of $0.5 M_\odot$ of $^{56}Fe$ in a galaxy with a volume of $\sim 3\times10^2$ kpc$^3$ and a mean density of gas of $\sim 1$ cm$^{-3}$. The current number density of iron is $\sim 10^{-5}$ (by mass $\sim 10^{-7}$) $n_H$. This gives a simple estimate of the minimum metallicity $(Z/Z_\odot)$ for a galaxy.

Question What is the mixing timescale(s) for material in the Galaxy, based on the processes we've already discussed.

Question Consider f to be a stochastic variable. Then $\dot{M}_g$ is a Langevin equation, well known from stochastic differential equations. Discuss some plausible lines of attack on a solution for $M_g(t)$.

NB The most recent (2000-present) studies of stellar abundances at about $10^{-4}-10^{-3} Z_\odot$ indicate a nearly pure r-process (rapid neutron capture) pattern for heavy elements ($A > 40$). This implies a possible single source, SNIa or related objects; also O/Fe ↓ with increasing Fe, and since this is also an indicator of supernova processing, it suggests $\Psi$ increased in the past and possibly a single source at lowest metallicities.

Question : What are the evolutionary consequences of Z increasing with time? This very general question summarizes much of our discussion of the past few weeks.

One last point. At the present epoch, in interstellar clouds, cooling is due to CO and driven by $H_2$ collisional excitation, leading to radiative far-IR and mm-emission. The cooling therefore depends on $Z^2$ for the formation of the CO. Now since you would expect in the earlier ~~ep~~ time that C and O are less abundant, this should increase T and, hence, $M_J$. This is the reason for originally thinking that $\Phi(m)$ is time dependent. But a small residual abundance of electrons leads to the formation of $H_3^+$ due to $H^-$ formation and $H_2$, so even at $Z \to 0$ there is still some residual dipole cooling through the ions.