The role played by the “de Sitter effect” in the rise of modern relativistic cosmology

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1 - Introduction

- Modern cosmology: study of the origin, structure and evolution of the universe as a whole, based on the interpretation of astronomical observations at different wave-lengths through the laws of physics

- The early years: 1917-1930. Extremely relevant period, characterized by <u>new ideas, discoveries, controversies</u>, in the light of the <u>first comparison</u> of theoretical world-models with observations at the turning point of relativity revolution

- Particular attention on the interest in the so-called "de Sitter effect", a theoretical redshift-distance relation which is predicted through the line element of the empty universe of de Sitter

- The de Sitter effect: fundamental influence in contributions that scientists as de Sitter, Eddington, Weyl, Lanczos, Lemaître, Robertson, Tolman, Silberstein, Wirtz, Lundmark, Stromberg, Hubble offered during the 1920's both in theoretical and in observational cosmology (emergence of the modern approach of cosmologists)

➔ *Predictions and confirmations of an appropriate redshift-distance relation marked the tortuous process towards the change of viewpoint from the 1917 paradigm of a static universe to the 1930 picture of an expanding universe evolving both in space and in time*

2 - 1917:
the Universes of General Relativity

- General relativity: a suitable theory in order to describe the whole of space, time and gravitation?
- Two different metrics, i.e. two “rival” solutions of relativistic field equations, to achieve, respectively, the requirement of the relativity of inertia (EU) and the postulate of general covariance (dSU)
- 1916 – 1918: Einstein – de Sitter debate

✓ EU: static, finite and unbounded universe filled by matter (spherical model + “cosmic time”)

✓ dSU: static, finite and completely empty universe (hypersphere or hyperboloid)

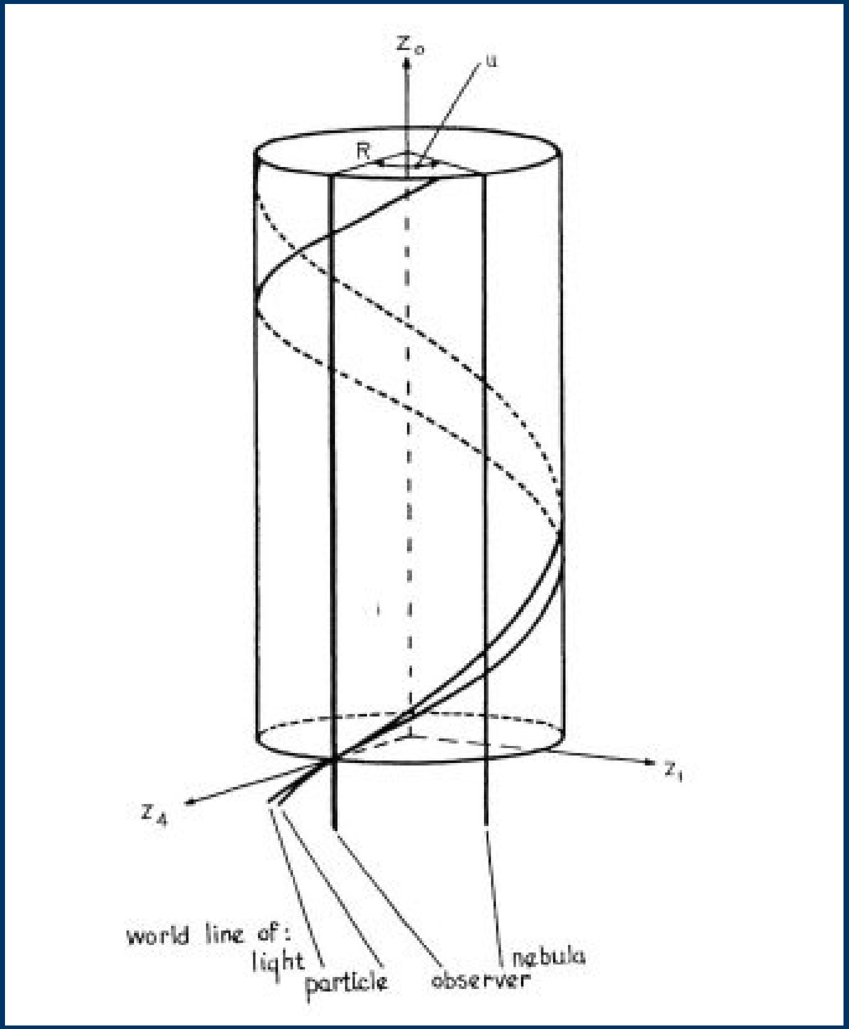
Einstein and the relativity of inertia

- Mach's influence: only relative motions existed, and had to be referred directly to all masses in the universe, not to an absolute space
- "In a consistent theory of Relativity there can be no inertia *relatively to 'space'*, but only an inertia of masses, *relatively to one another*"
- Fundamental question: the metric should be fully determined by matter. General relativity should have expressed that there was neither locally nor globally any independent property of space.

➔ Thus the universe as a whole represented to Einstein an ideal setting in which the concept of inertia and its relativity could be verified.

A "finite and yet unbounded universe"

- Problem: set of potentials invariant for all transformations at infinity
- Solution: no boundary conditions! Condition of closure: spherical model of the universe



From: Robertson 1933

- ✓ Hypothetical average density of matter, uniformly and homogeneously distributed through space
- ✓ Neither privileged positions nor preferred directions

$$g_{\alpha\beta} = -\left[\delta_{\alpha\beta} + \frac{x_\alpha x_\beta}{R^2 - \left(x_1^2 + x_2^2 + x_3^2\right)}\right]$$

- Static nature of stellar system: both spatial terms of the metric and the curvature radius were not depending on time

$$ds^2 = dx_4^2 - g_{\alpha\beta}\, dx_\alpha\, dx_\beta$$

- “The odd thing is that now a quasi-absolute time and a preferred coordinate system do reappear in the end, while fully complying the requirements of relativity”

- *This model fully achieved the relativity of inertia. There was not any independent property of space which claimed to the origin of inertia, so the latter was entirely produced by all masses in the universe (1918: Mach's Principle).*

The cosmological constant

- Condition of spatial closure: both the gravitational potential and the hypothetical average density of ponderable matter remained constant *in space*.
- Cosmological constant? Accounting for the supposed static nature of the universe, i.e. to preserve the potential and the density of matter constant *in time*. It balanced gravitational effects on large scale.

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R-\lambda g_{\mu\nu}=-k\,T_{\mu\nu}$$

$$\lambda=\frac{k\,\rho\,c^2}{2}=\frac{1}{R^2}$$

- "Cosmological Considerations in the General Theory of Relativity", February 1917: "first serious proposal for a novel topology of the universe". The geometry of the universe as a whole could be described by relativistic field equations.

The universe of de Sitter

- EU: "material postulate of relativity of inertia" (i.e. it existed only by the presence of world-matter)
- Time coordinate in EU: nothing else than an <u>absolute time</u>
- World-matter: replaced <u>absolute space</u> of Newtonian theory.

At the hypothetical value $R \longrightarrow \infty$, the whole of $g_{\mu\nu}$ proposed by Einstein degenerated to:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1 \end{pmatrix}$$

This set of values was invariant for all transformations for which, at infinity, $t' = t$.

- Postulate that at infinity all $g_{\mu\nu}$ were invariant for all transformations: "If at infinity all $g_{\mu\nu}$ were *zero,* then we could truly say that the whole of inertia, as well as gravitation, is thus produced. This is the reasoning which has led to the postulate that at infinity all $g_{\mu\nu}$ shall be zero"

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

- "Mathematical relativity condition" or equivalently the "mathematical postulate of relativity of inertia": "the world as a whole can perform random motions without us (within the world) being able to observe it. (...) The postulate of the invariance of the $g_{\mu\nu}$ at infinity has no physical meaning. It is purely mathematical".

A universe without "world matter"

- Metric proposed by de Sitter: solution of field equations also maintaining the "undeterminable and undesirable" λ-term, but without matter

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \lambda g_{\mu\nu} = 0$$

- Neither universal time, nor differences between "time"-coordinate and other coordinates: none of these coordinates had any physical meaning.
- Value of the curvature radius determined by the cosmological constant:

$$\lambda = \frac{3}{R^2}$$

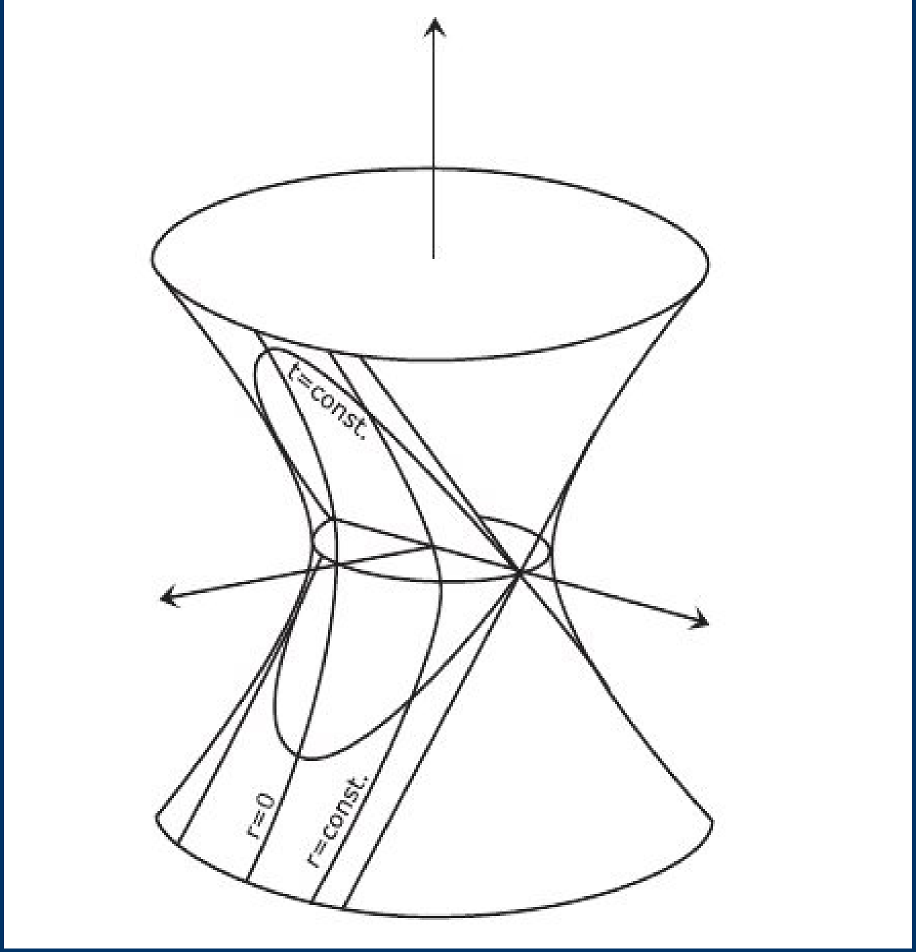
- *4-D* hyper-sphere embedded in a *5-D* Euclidean space

equivalently

- a *(3+1)-D* hyperboloid embedded in a *(4+1)-D* Minkowski space-time.

$$g_{\mu\nu}=-\left[\delta_{\mu\nu}+\frac{x_\mu x_\nu}{R^2-\left(x_1^2+x_2^2+x_3^2+x_4^2\right)}\right]$$

$$ds^2=\frac{c^2t^2-dx^2-dy^2-dz^2}{\left[1-\frac{\lambda}{12}\left(c^2t^2-x^2-y^2-z^2\right)\right]^2}$$



From: Lord 1974

solution A vs solution B

- By spherical polar coordinates: hyperboloid universe (model B) as the Einstein universe (model A), i.e. as a *3-D* <u>hyper-sphere</u> embedded in a *4-D* Euclidean space:

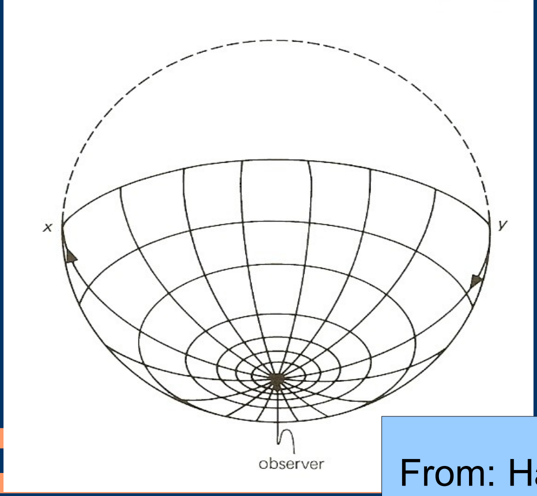
A

$$ds_A^2 = -dr^2 - R^2 \sin^2 \frac{r}{R} (d\psi^2 + \sin^2 \psi \, d\theta^2) + c^2 dt^2$$

B

$$ds_B^2 = -dr^2 - R^2 \sin^2 \frac{r}{R} (d\psi^2 + \sin^2 \psi \, d\theta^2) + \boxed{\cos^2 \frac{r}{R} c^2 dt^2}$$

- Elliptical geometry: space has constant curvature but no "anti-Sun" (i.e. no antipodal points).



From: Harrison 2000

Geometry of de Sitter universe

- "Static form" (hyperspherical coordinates):

$$ds^2 = -dr^2 - R^2 \sin^2 \frac{r}{R}(d\psi^2 + \sin^2 \psi d\theta^2) + \cos^2 \frac{r}{R}\, c^2 dt^2.$$

- Projection on Minkowski space-time (hyperboloid; pseudo-spherical coordinates):

$$ds^2 = -\frac{d\,\mathbf{r}^2}{\left(1+\frac{\mathbf{r}^2}{R^2}\right)^2} - \frac{\mathbf{r}^2[d\psi^2 + \sin^2\psi d\theta^2]}{1+\frac{\mathbf{r}^2}{R^2}} + \frac{c^2dt^2}{1+\frac{\mathbf{r}^2}{R^2}}.$$

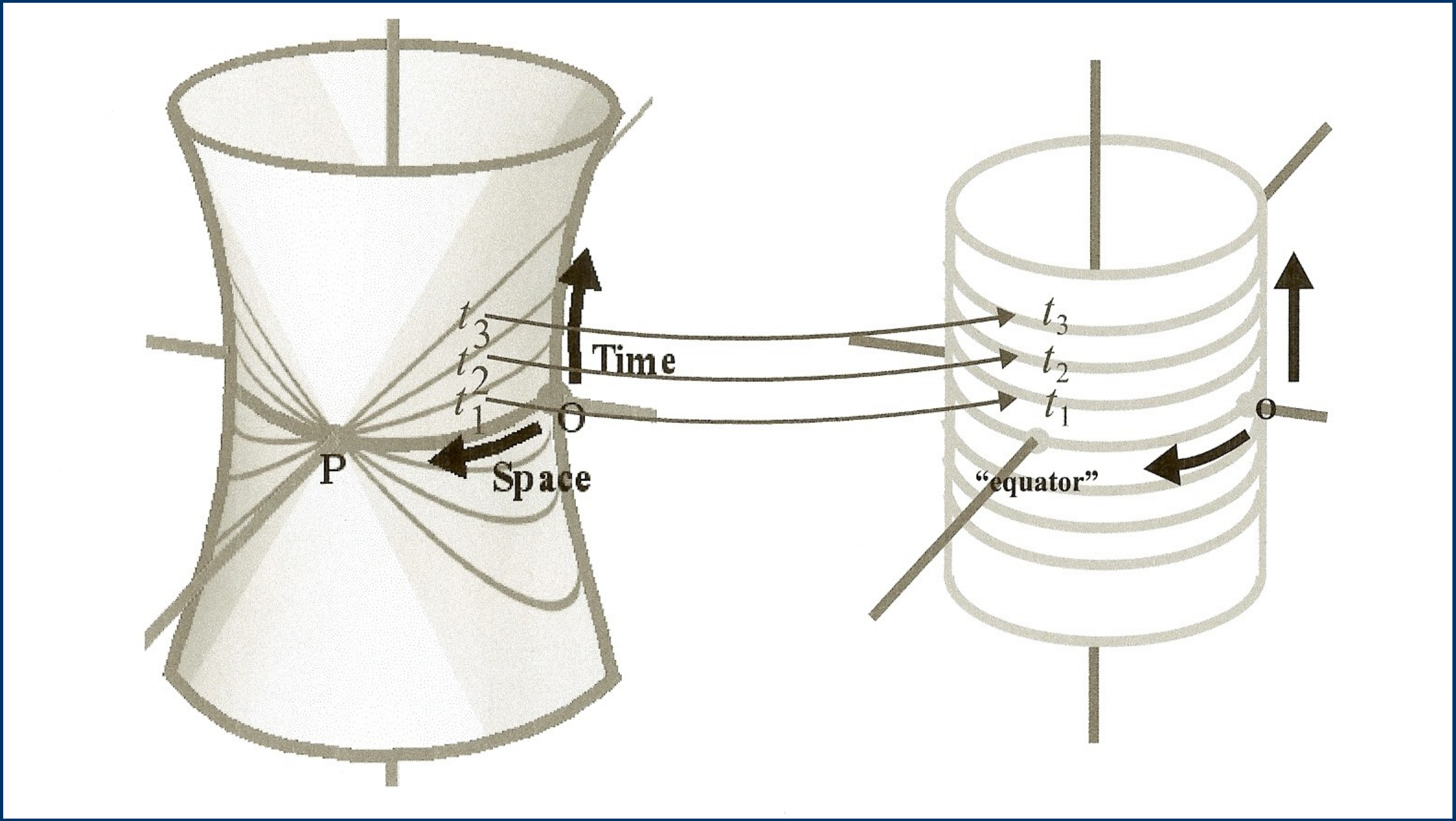
- Why several formulations of dS line element? dSU is a space-time of constant curvature; there is not a unique choice to specify the 4-velocity which represents the average motion of test particles.

Einstein's criticism

- Einstein “must have been disappointed”. Objection: the coefficient of the time-coordinate term in system B depended on position. At the surface $r = (\pi/2)R$ time-potential was = 0. Time clocks slowed down approaching this “equator”: singularity.
- “The de Sitter system does not look at all like a world free of matter, but rather a world whose matter is concentrated entirely on the surface $r = (\pi/2)R$”
- 1918: the issue is solved by Felix Klein: coordinate singularity, removed by using hyperboloid coordinates

Therefore the matter-free model proposed by de Sitter was free of singularities, and its space-time points were all equivalent.

- Einstein: dS “anti-Machian” universe was not a physical possibility. It was not “static”: hyper-surfaces at different times intersected each-other at the equator, so that time-coordinate could not be uniquely defined.



From: Janssen 2005

Static? Stationary? Expanding?

- de Sitter (1917): static and empty universe
- Eddington (1923), Silberstein (1924), Tolman (1929): static dSU
- Lanczos (1924): static if "the coefficients of the metric are independent of time in a coordinate system in which all masses are at rest on average". dSU (1917) static but not stationary
- Weyl (1923+1930), Robertson (1928), Lemaitre (1925): "the fundamental world-lines expand away from each other, but they also present the same appearance at any cosmic time". dSU stationary but not static [Retrospect: expanding dSU].
- Friedmann (1922), Lemaitre (1927): curvature radius depending on time $R \equiv R(t) = a(t)$. Expanding EU

3 - The “de Sitter effect”

- 1917: de Sitter related spectral shifts to velocity and distance of astronomical objects (B-stars and spiral nebulae) by his own relativistic solution
- During the 1920's: several scientists dealt with properties of dSU and proposed different formulations of redshift (velocity) - distance relation. Despite its lack of matter, the empty and “non-static” dSU was preferred to the “rival” static EU just because of its astronomical consequences
- Up to 1930 (before the expanding universe): dS Effect was the only possible, however puzzling, explanation of redshift problem

Redshift

$$z = \frac{\lambda_0 - \lambda_e}{\lambda_e} \Rightarrow 1 + z = \frac{\lambda_0}{\lambda_e},$$

- Gravitational shift (red or blue), due to light traveling close to massive bodies

$$z_G = \frac{1}{\sqrt{(1 - \frac{R_s}{R})}} - 1.$$

- Doppler shift (red or blue), originated by relative motions between objects through space

$$z_D = \frac{v}{c},$$

$$z_D = \left(\frac{c+v}{c-v}\right)^{1/2} - 1.$$

- Expansion (cosmological) redshift, due to waves stretched by the expansion (Lemaitre, 1927)

$$z_C = \frac{a_0}{a_e} - 1,$$

Redshift-distance relation 1917: de Sitter's first suggestion

- Question: Einstein model (A) or de Sitter model (B)?

- Radius system A? $R_A \approx 10^{12}$ AU

- Absorption (40 magnitudes)
- Apparent diameter
- Star density

- Radius system B? Redshift interpretation...

- dS line element: g_{44} depends on position. Frequency of light dimishes with increasing distances.
- One should expect displacements of spectral lines towards the red: "spurious radial velocity" produced by the inertial field
- Observation of B stars:

$$K = 0.634\, M^{\frac{2}{3}}\, \rho^{\frac{1}{3}}.$$

K term (Campbell, 1911) = +4.5 km/sec

Since +1.5 km/sec was actually due to gravitational redshift, the remaining part was due to the inertial field

$$f = g_{44} = 1 + \gamma \simeq 1 - 2\frac{v}{c} = 1 - 2 \cdot 10^{-5}.$$

- Radius of system B?

if r (B stars) = $3 \cdot 10^{7}$ AU, then $R_B = 2/3 \cdot 10^{10}$ AU

N.G.C. 4594	Pease	+ 1180	km/sec
	Slipher	+ 1190	"
N.G.C. 1068	Pease	+ 765	km/sec
	Slipher	+ 1100	"
	Moore	+ 910	"
Andromeda	Wright	− 304	km/sec
	Pease	− 329	"
	Slipher	− 300	"

By using radial velocities of 3 nebulae, and R obtained through the previous relation, it followed a distance for these objects of about

r = 4.10^8 AU

(with average v=+ 600 km/sec)

"gravitational" contribution to the de Sitter effect
- spurious velocity -

$$ds^2 = -dr^2 - R^2 \sin^2 \frac{r}{R}(d\psi^2 + \sin^2 \psi d\theta^2) + \cos^2 \frac{r}{R} c^2 dt^2.$$

$$z \equiv \frac{\lambda_0 - \lambda_e}{\lambda_e} = \frac{1}{\sqrt{g_{44}}} - 1 = \sec \frac{r}{R} - 1.$$

$$ds^2 = -\frac{d\,\mathbf{r}^2}{\left(1 + \frac{\mathbf{r}^2}{R^2}\right)^2} - \frac{\mathbf{r}^2[d\psi^2 + \sin^2 \psi d\theta^2]}{1 + \frac{\mathbf{r}^2}{R^2}} + \frac{c^2 dt^2}{1 + \frac{\mathbf{r}^2}{R^2}}.$$

$$\frac{\lambda_0 - \lambda_e}{\lambda_e} \simeq \frac{1}{2} \frac{\mathbf{r}^2}{R^2}.$$

Geodesics

$$\frac{d^2\mathbf{r}}{c^2dt^2} = \frac{\mathbf{r}}{R^2} + \mathbf{r}\left[\left(\frac{d\psi}{c\,dt}\right)^2 + \sin^2\psi\left(\frac{d\theta}{c\,dt}\right)^2\right],$$

$$\frac{d^2\theta}{c^2dt^2} = -\frac{2}{\mathbf{r}}\left(\frac{d\mathbf{r}}{c\,dt}\right)\left(\frac{d\theta}{c\,dt}\right) - 2\cot\psi\left(\frac{d\psi}{c\,dt}\right)\left(\frac{d\theta}{c\,dt}\right),$$

$$\frac{d^2\psi}{c^2dt^2} = -\frac{2}{\mathbf{r}}\left(\frac{d\mathbf{r}}{c\,dt}\right)\left(\frac{d\psi}{c\,dt}\right) + \sin\psi\cos\psi\left(\frac{d\theta}{c\,dt}\right)^2.$$

$$\left(\frac{d\mathbf{r}}{d\theta}\right)^2 = \frac{\mathbf{r}^2(\mathbf{r}^2 - a^2)(\mathbf{r}^2 + b^2)}{a^2b^2}.$$

Orbit: hyperbola

Velocity

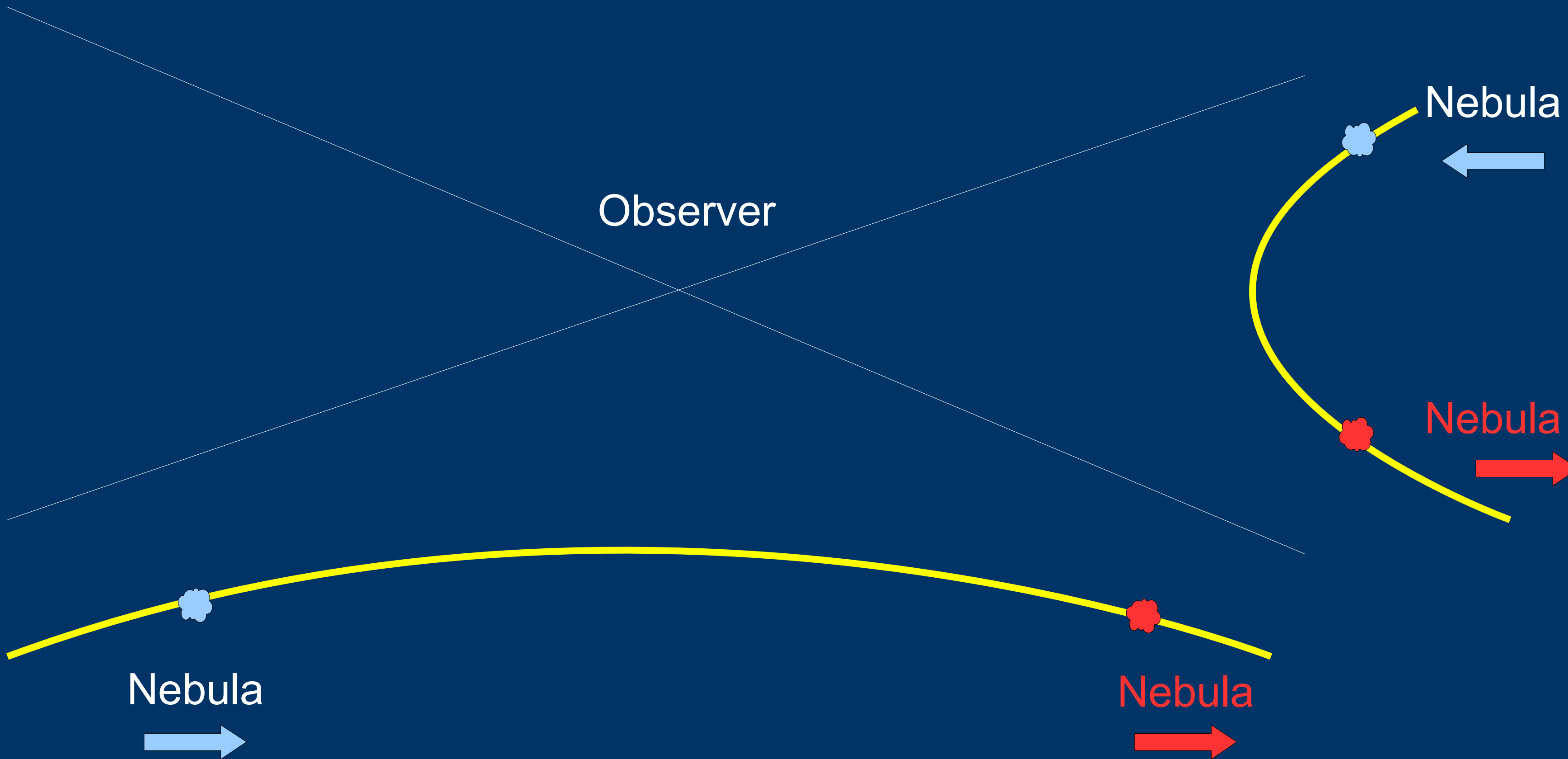
$$\left(\frac{d\mathbf{r}}{dt}\right)^2 = \frac{\mathbf{r}^2}{R^2}\left(1 - \frac{a^2}{\mathbf{r}^2}\right)\left(1 + \frac{b^2}{\mathbf{r}^2}\right).$$

de Sitter effect

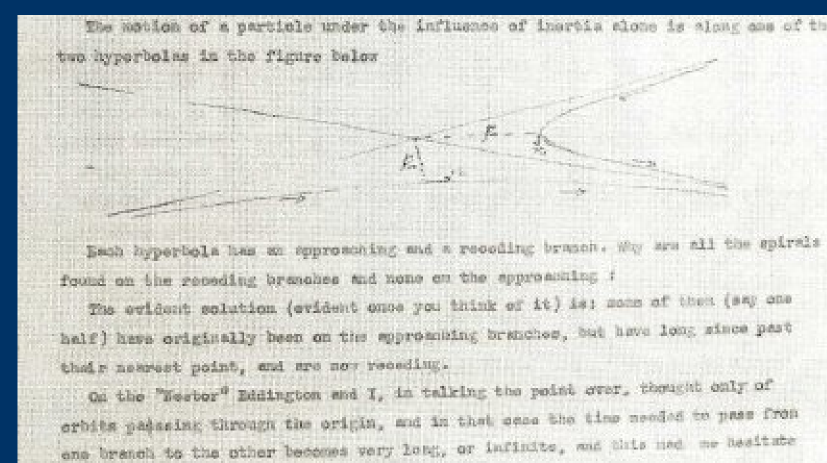
$$\frac{\lambda_0 - \lambda_e}{\lambda_e} \simeq \pm\frac{\mathbf{r}}{R} + \frac{1}{2}\left(\frac{\mathbf{r}}{R}\right)^2 .$$

"If continued observations should confirm the fact that spiral nebulae
have sistematically positive radial velocities,
this would be certainly an indication to adopt
system B in preference to A" (1917)

- de Sitter: inaugurated the attempts to relate astronomical observations to the geometry of the universe
- Director (Leiden Observatory) from 1919 to 1934.
- From 1920 to 1930: no papers about cosmology.



“Why are all the spirals found on the receding branches?
Some of them have originally been on the approaching branches,
but have long since past their nearest point, and are now receding”



The motion of a particle under the influence of inertia alone is along one of the two hyperbolas in the figure below

Each hyperbola has an approaching and a receding branch. Why are all the spirals found on the receding branches and none on the approaching ?

The evident solution (evident once you think of it) is: some of them (say one half) have originally been on the approaching branches, but have long since past their nearest point, and are now receding.

On the "Nestor" Eddington and I, in talking the point over, thought only of orbits passing through the origin, and in that case the time needed to pass from one branch to the other becomes very long, or infinite, and this made me hesitate

Letter from de Sitter to Schlesinger (1929)
[Courtesy: Leiden Archives]

Matter or motion? Eddington analysis

- EU objectionable for the presence of the world-matter (which recalled the aether)
- dSU “much less open to objection”, because it offered the possibility to explain large velocities of spirals

- Suggestion: actual universe as an intermediate state between EU and dSU

...the 1927 expanding model by Lemaître would have represented such an intermediate solution

By the new coordinate $\tilde{r}$ defined as:

$$\tilde{r} = R \sin \chi,$$

the line element of system B became:

$$ds^2 = -\frac{1}{\left(1 - \frac{\tilde{r}^2}{R^2}\right)} d\tilde{r}^2 - \tilde{r}^2 d\theta^2 - \tilde{r}^2 \sin^2 \theta \, d\phi^2 + \left(1 - \frac{\tilde{r}^2}{R^2}\right) dt^2.$$

Geodesics

$$\frac{d^2 \tilde{r}}{ds^2} = \frac{1}{3} \lambda \, \tilde{r},$$

"A particle at rest will not remain at rest unless it is at the origin".
Tendency of particles to scatter, so that dSU became non-static as soon as any matter was inserted in it

de Sitter effect? "during the time light is traveling to us, the nebula is being accelerated by the cosmical repulsion, and acquires an additional outward velocity".

The velocity was **spurious** at the time of emission, and became **genuine** at the time of observation.

$$z \simeq \frac{1}{2} \left(\frac{\tilde{r}}{R}\right)^2,$$

Radial Velocities of Spiral Nebulae

+ indicates receding, – approaching

N. G. C.	R. A. h m	Dec. ° ′	Rad. Vel. km. per sec.
221	0 38	+40 26	− 300
224*	0 38	+40 50	− 300
278†	0 47	+47 7	+ 650
404	1 5	+35 17	− 25
584†	1 27	− 7 17	+1800
598*	1 29	+30 15	− 260
936	2 24	− 1 31	+1300
1023	2 35	+38 43	+ 300
1068*	2 39	− 0 21	+1120
2683	8 48	+33 43	+ 400
2841†	9 16	+51 19	+ 600
3031	9 49	+69 27	− 30
3034	9 49	+70 5	+ 290
3115†	10 1	− 7 20	+ 600
3368	10 42	+12 14	+ 940
3379*	10 43	+13 0	+ 780
3489†	10 56	+14 20	+ 600
3521	11 2	+ 0 24	+ 730
3623	11 15	+13 32	+ 800
3627	11 16	+13 26	+ 650
4111†	12 3	+43 31	+ 800
4151*	12 6	+39 51	+ 980
4214	12 12	+36 46	+ 300
4258	12 15	+47 45	+ 500
4382†	12 21	+18 38	+ 500
4449	12 24	+44 32	+ 200
4472	12 25	+ 8 27	+ 850
4486†	12 27	+12 50	+ 800
4526	12 30	+ 8 9	+ 580
4565†	12 32	+26 26	+1100
4594*	12 36	−11 11	+1100
4649	12 40	+12 0	+1090
4736	12 47	+41 33	+ 290
4826	12 53	+22 7	+ 150
5005	13 7	+37 29	+ 900
5055	13 12	+42 37	+ 450
5194	13 26	+47 36	+ 270
5195†	13 27	+47 41	+ 240
5236†	13 32	−29 27	+ 500
5866	15 4	+56 4	+ 650
7331	22 33	+33 23	+ 500

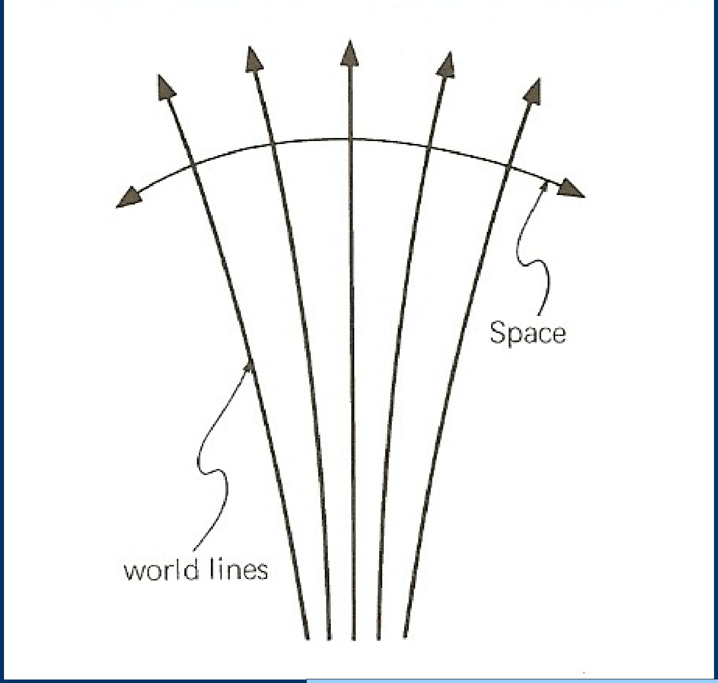
Slipher 1922

From: The Mathematical Theory of Relativity, Eddington 1923

Redshift according to Weyl

- dSU (hyperboloid): light cones do not overlap themselves in the future direction
- Weyl Principle (1923): nebulae have world lines diverging towards the future, and are stationary in a space perpendicular to the world lines.

Nebulae recede one to another
with apparent velocities
which increase with their separation



From: Harrison 2000

- Unique definition of spectral displacements: a periodic process at the source was periodic also for the observer, however with a period increased by the ratio of the observer and source proper time:

$$\alpha = \frac{d\sigma}{ds}.$$

- The relation was linear at small distances:

$$z = \frac{\lambda_0 - \lambda e}{\lambda_e} = \alpha - 1,$$

$$z = \tan \frac{d}{R}$$

Lanczos on de Sitter's universe

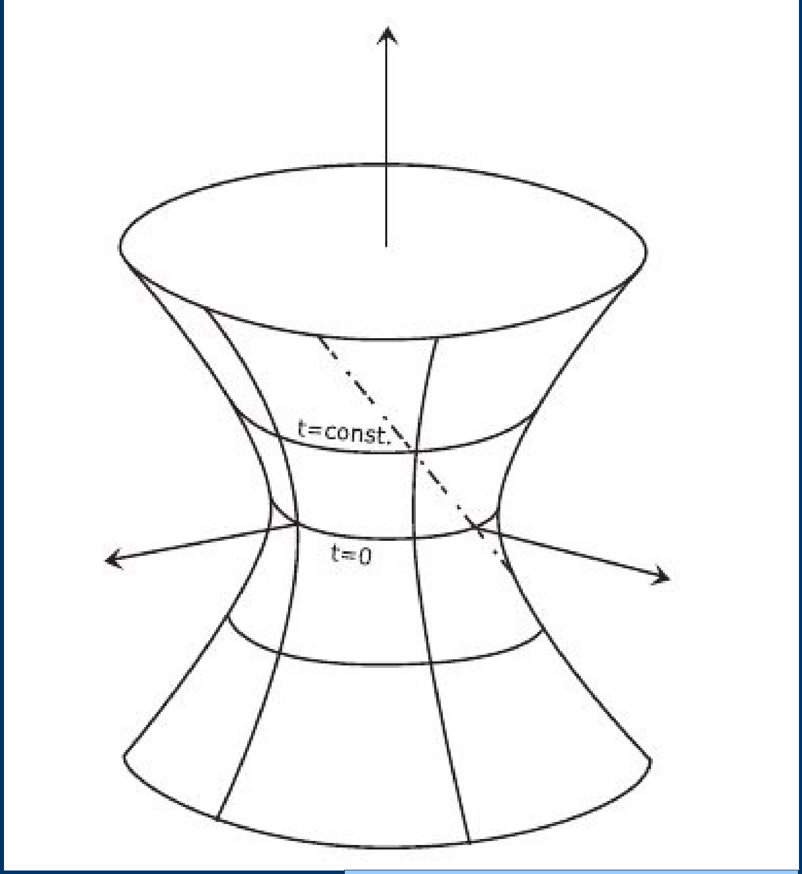
- 1922: $$ds^2 = -\frac{(e^t + e^{-t})^2}{4}(d\phi^2 + \cos^2\phi\, d\psi^2 + \cos^2\phi\cos^2\psi\, d\chi^2) + dt^2.$$

- 1923: No gravitational shifts in dSU

$$z = \cos\frac{a}{R} - \sin\frac{a}{R}\sinh\tau_0 - 1,$$

Approximate linear relation (distance)

Time-dependent additional term



From: Lord 1974

Silberstein's contributions (1924)

Silberstein criticized at polemical level the **general recession** predicted by Weyl and Eddington

Globular clusters
rather than nebulae
(Positive and negative velocities)

“Small” curvature radius

$$z = \gamma\left[1 \pm \sqrt{1 - \frac{\cos^2\sigma}{\gamma^2}}\right] - 1 = \gamma\left[1 \pm \frac{v}{c}\sec\sigma\right] - 1.$$

$$z = \pm\sin\sigma = \pm\frac{v}{c}\sec\sigma \simeq \pm\frac{r}{R}.$$

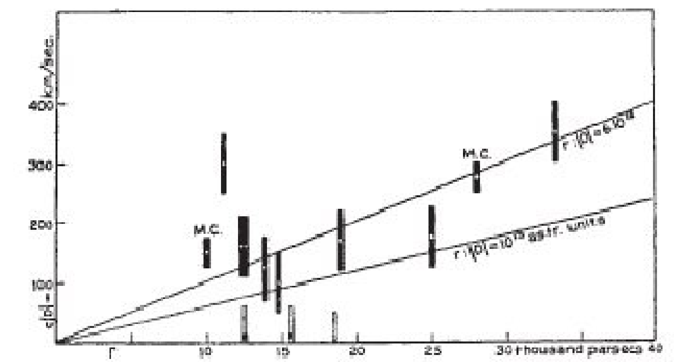


FIG. 1.

Near objects:
Special relativistic Doppler effect

Distant objects:
linear effect

- Curvature radius: R = 6 $\cdot$ 10^{12} AU, twice the value that Shapley proposed in 1919 for the Milky Way as a "unique" galactic system

- In 1929 Tolman took into account Silberstein results, and proposed a general formulation of Doppler shifts (both receding and approaching).
- dSU "static form" could not unmistakably explain redshifts of galaxies

$$z = \frac{k \pm \sqrt{k^2 - 1 + \frac{\tilde{r}^2}{R^2} - \frac{h^2}{\tilde{r}^2} + \frac{h^2}{R^2}}}{1 - \frac{\tilde{r}^2}{R^2}} - 1.$$

Lemaitre's 1925 notes

- non-static line element of dSU in order to avoid the singularity at the mass-horizon.
- dSU had to be abandoned not because it was not static,

but because it "became" an infinite universe (and not a finite one)

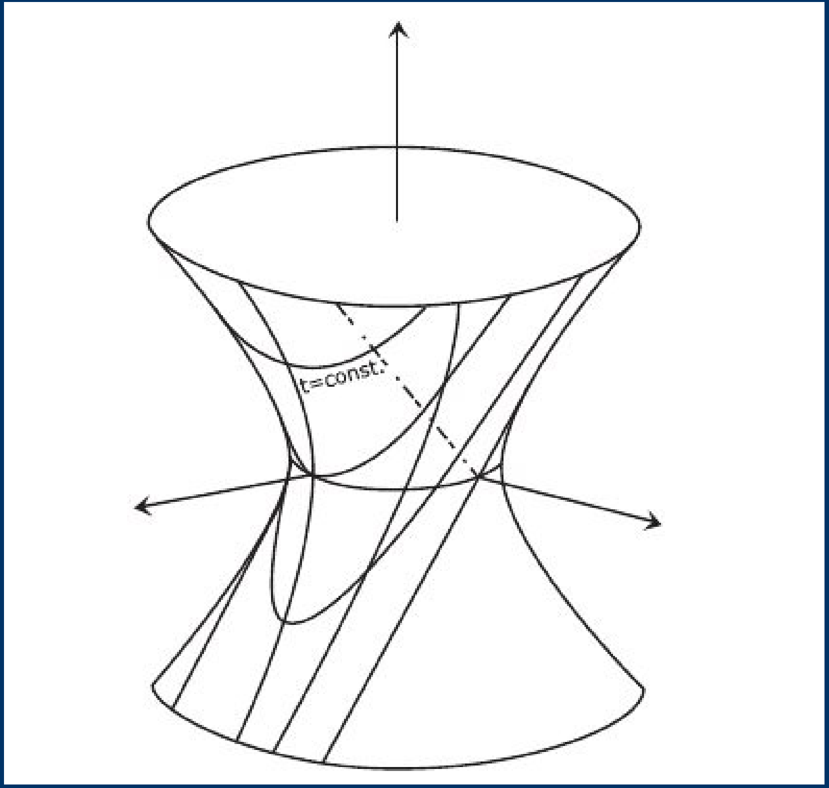
$$\chi = \arcsin \frac{r}{t},$$

$$\tau = \frac{1}{2} \ln(t^2 - r^2),$$

$$ds^2 = R^2 \frac{-dx^2 - dy^2 - dz^2 + dt^2}{t^2}$$

$$T = \pm \int \frac{dt}{t} = \pm \ln t,$$

$$ds^2 = R^2[-e^{\pm 2T}(dx^2 + dy^2 + dz^2) + dT^2]$$



Same Doppler shift as Silberstein, however with a unique sign

$$\frac{\Delta\lambda}{\lambda_0} = \frac{\lambda_e - \lambda_0}{\lambda_0} = \frac{dt_e}{dt_0} - 1 = \frac{t_e}{t_0} - 1 = -\frac{r}{t_0} = -\sin\chi.$$

Robertson and a linear relation (1928)

$$\rho = r\, e^{kct},$$

$$\tau = t - \frac{1}{2kc}\log(1 - k^2 r^2 e^{2kct}).$$

$$ds^2 = -e^{2kct}(dr^2 + r^2 d\theta^2 + r^2 \sin^2 d\phi^2) + c^2 dt^2$$

This exponential form of dSU was later used in STEADY STATE cosmology, and is at present used for ENERGY VACUUM dominated universes

Doppler effect

$$\frac{\Delta\lambda}{\lambda_e} \equiv \frac{\lambda_0 - \lambda_e}{\lambda_e} = \frac{dt_0}{dt_e} - 1 = \frac{kl_e}{1 - kl_e}.$$

$$v \simeq c\frac{l}{R}.$$

Linear relation (at small distances)
confirmed by 1926 Hubble data (for distances)
and 1923 Slipher data (for velocities)

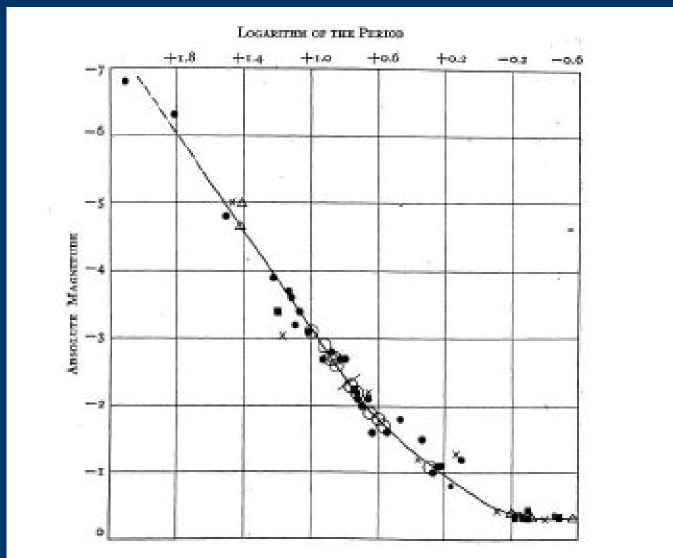
Summary of different redshift-distance relations

	frame	redshift (velocity)-distance relation
de Sitter, 1917	static	$z = \frac{v}{c} \simeq \pm\frac{\mathrm{r}}{R} + \frac{1}{2}\left(\frac{\mathrm{r}}{R}\right)^2$
Eddington, 1923	static	$z \simeq \frac{1}{2}\left(\frac{\bar{r}}{R}\right)^2$
Weyl, 1923 + 1930	stationary, $k = 0$	$z = \tan\frac{d}{R}$
Lanczos, 1923	stationary, $k = +1$	$z = \cos\frac{a}{R} - \sin\frac{a}{R}\sinh\tau_0 - 1$
Silberstein, 1924	static	$z = \pm\sin\sigma = \pm\frac{v}{c}\sec\sigma \simeq \pm\frac{r}{R}$
Lemaître, 1925	stationary, $k = 0$	$z = \sin\chi$
Robertson, 1928	stationary, $k = 0$	$z \simeq \frac{l}{R}$
Tolman, 1929	static	$z = \left(k \pm \sqrt{k^2 - 1 + \frac{\bar{r}^2}{R^2} - \frac{h^2}{\bar{r}^2} + \frac{h^2}{R^2}}\right) / \left(1 - \frac{\bar{r}^2}{R^2}\right) - 1$

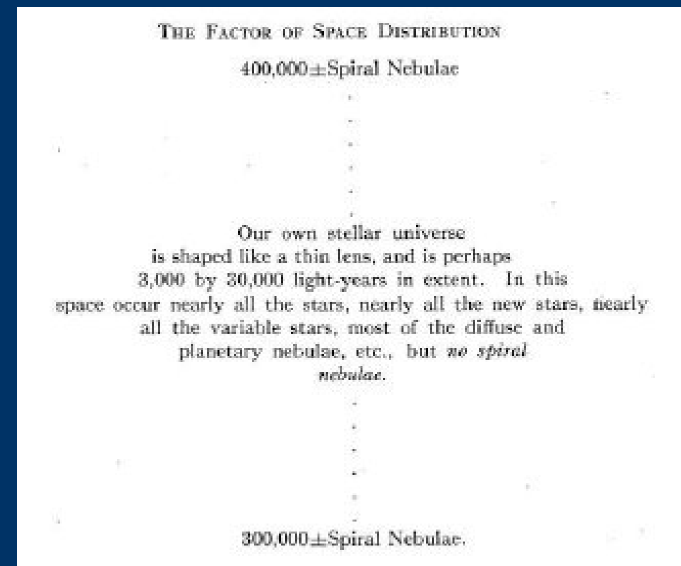
The rise of observational cosmology

- Structure of our Galaxy: Kapteyn's universe (1920-1922). By statistical approach: flat rotating disk, with the Sun at 650 pc from the center
- Fundamental step in the cosmic distance ladder: Period-Luminosity relation (Leavitt 1913). <u>Cepheid stars as distance indicators</u>
- Shapley (1918-1919): Milky Way as a flat rotating disk (diameter 300'000 light years), surrounded by a spherical halo of globular clusters
- Question: <u>nature of the nebulae</u>? Spectroscopy revealed that spirals have the same features of stellar systems: “island universes”?

- Slipher (since 1912) + Humason (since 1927): relevant velocities of spirals
- Great Debate (1920): Shapley vs Curtis



Shapley



Curtis

- Hubble (1925): Cepheids in M31+ M33 (285'000 pc), NGC 6822 (214'000 pc). Spirals are truly extragalactic systems!

1924: Wirtz and de Sitter's cosmology

K term
Stars: Campbell (1911)
Nebulae: Paddock (1916)

$$V = X \cos\alpha \cos\delta + Y \sin\alpha \cos\delta + Z \sin\delta + K.$$

V= velocity of nebula (α,δ); Sun (-X, -Y, -Z)

$$v = 914 - 479 \cdot \log Dm,$$

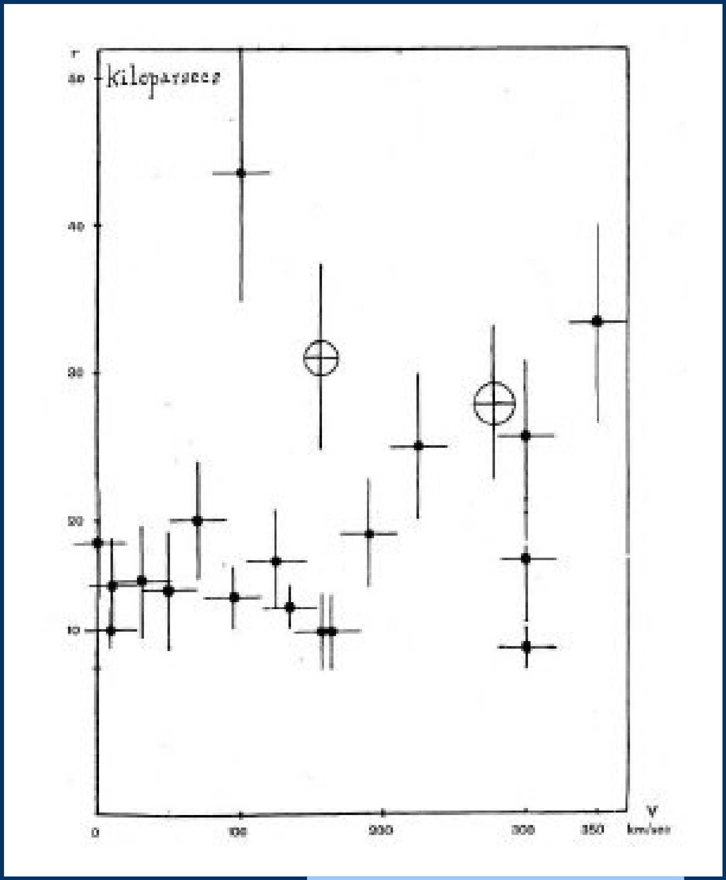
$$\log Dm = 0.96 - 0.000432 \cdot v.$$

$$v = +574 \text{ km/sec}$$

Wirtz considered
apparent diameter (Dm)
of 29 nebulae,
(for which
K = 840 ± 140 km/sec)

... no distances before Hubble (1925)!

Astronomers at work: Lundmark and Stromberg



Lundmark 1924

- Lundmark (1924) denied the correctness of Silberstein result (for the choice of globular clusters).
- Novae in M31: 200.000 pc = basic unit for distance scale of 44 spirals (constant diameters and constant absolute magnitudes)
- “not a very definite” relation

- Stromberg (1925) studied velocities of 43 spirals, the Magellanic Clouds and 18 globular clusters

"No sufficient reason to believe that there exists any dependence of radial motion upon distance from the Sun"

- Lundmark (1925): solar motion with the K term in the form:

$$K = 513 + 10.365\,r - 0{\cdot}047r^2\,\mathrm{km\,s^{-1}}.$$

Negative term: upper limit to the velocities of spirals

RADIAL VELOCITIES OF GLOBULAR CLUSTERS AND NON-GALACTIC NEBULAE

N.G.C.	α (1900)	δ (1900)	V Slipher	V Others	V Mean
			km/sec.	km/sec.	km/sec.
221	0^h37^m2	+40° 19′	− 300	...	...
224	0 37	+40 43	− 300	− 320*	−315
278	0 46.4	+47 1	+ 650::	...	...
404	1 3.9	+35 11	− 25	...	...
584	1 26.3	− 7 23	+1800:	...	...
598	1 28.2	+30 9	− 260	− 70†	− 70
936	2 22.5	− 1 36	+1300	...	...
1023	2 34.1	+38 38	+ 300	...	...
1068	2 37.6	− 0 26	+1120	+ 814‡	+ 916
1700	4 52.0	− 5 1	...	+ 800§	...
2681	8 46.4	+51 41	...	+ 700‖	...
2683	8 46.5	+33 48	+ 400	...	...
2841	9 15.1	+51 24	+ 600:	...	...
3031	9 47.4	+69 32	− 30	...	...
3034	9 47.6	+70 10	+ 290	...	...
3115	10 0.3	− 7 14	+ 600	...	...
3368	10 41.5	+12 21	+ 940	...	...
3379	10 42.6	+13 7	+ 780	+ 845§	+ 812
3489	10 55.0	+14 26	+ 600::	...	...
3521	11 0.7	+ 0 30	+ 730	...	...
3623	11 13.7	+13 38	+ 800	...	...
3627	11 15.0	+13 32	+ 650	...	...
4111	12 2.0	+43 37	+ 800:	...	...
4151	12 5.5	+39 58	+ 980	+ 940¶	+ 950
4214	12 10.6	+36 53	+ 300	...	...
4258	12 14.0	+47 52	+ 500	...	...
4382	12 20.3	+18 45	+ 500:	...	...
4449	12 23.3	+44 39	+ 200	...	...
4472	12 24.7	+ 8 33	+ 850	...	...
4486	12 25.6	+12 57	+ 800::	...	...
4526	12 29.0	+ 8 15	+ 580	...	...
4565	12 31.4	+26 32	+1100:	...	...
4594	12 34.8	−11 4	+1100	+1180**	+1140
4649	12 38.6	+12 6	+1090	...	...
4736	12 46.2	+41 40	+ 290	...	...
4826	12 51.8	+22 14	+ 150	...	...
5005	13 6.3	+37 36	+ 900	...	...
5055	13 11.3	+42 34	+ 450	...	...
5194	13 25.7	+47 43	+ 270	...	...
5195	13 25.7	+47 43	+ 240:	...	...
5236	13 31.4	−29 21	+ 500:	...	...
5866	15 3.7	+56 9	+ 650	...	...
7331	22 32.5	+33 54	+ 500	...	...
Magellanic	5 27	−68	...	+ 287††	...
clouds	1 6	−73 44	...	+ 168††	...
1851	5 10.8	−40 9	+ 315	...	...
1904	5 20.1	−24 37	+ 235	...	...
5024	13 8.0	+18 42	− 170	− 200‡‡	− 180
5272	15 37.6	+28 53	− 125	− 140‡‡	130
5904	15^h13^m5	+ 2° 27′	+ 10	...	...
6093	16 11.1	−22 41	+ 70	...	...
6205	16 38.1	+36 39	− 300	− 230§§	− 265
6218	16 42.0	− 1 46	...	+ 160‡‡	...
6229	16 44.2	+47 42	− 100:	...	...
6266	16 54.8	−29 58	+ 50	...	...
6273	16 56.4	−26 7	+ 30	...	...
6333	17 13.3	−18 25	+ 225	...	...
6341	17 14.1	+43 15	− 160	...	...
6626	18 18.4	−24 55	0	...	...
6934	20 29.3	+ 7 4	− 350:	...	...
7078	21 25.2	+11 44	− 95	− 93‡‡	− 94
7089	21 28.3	− 1 16	− 10	...	...
7099	21 34.7	−23 38	− 125	...	...

Stromberg 1925

1929: Hubble's law

$$V = X \cos\alpha \cos\delta + Y \sin\alpha \cos\delta + Z \sin\delta + kr.$$

46 nebulae
(24+22)

Redshift-distance
- empirical -
zc=Kr

Velocity-distance
- theoretical -
V=Hd

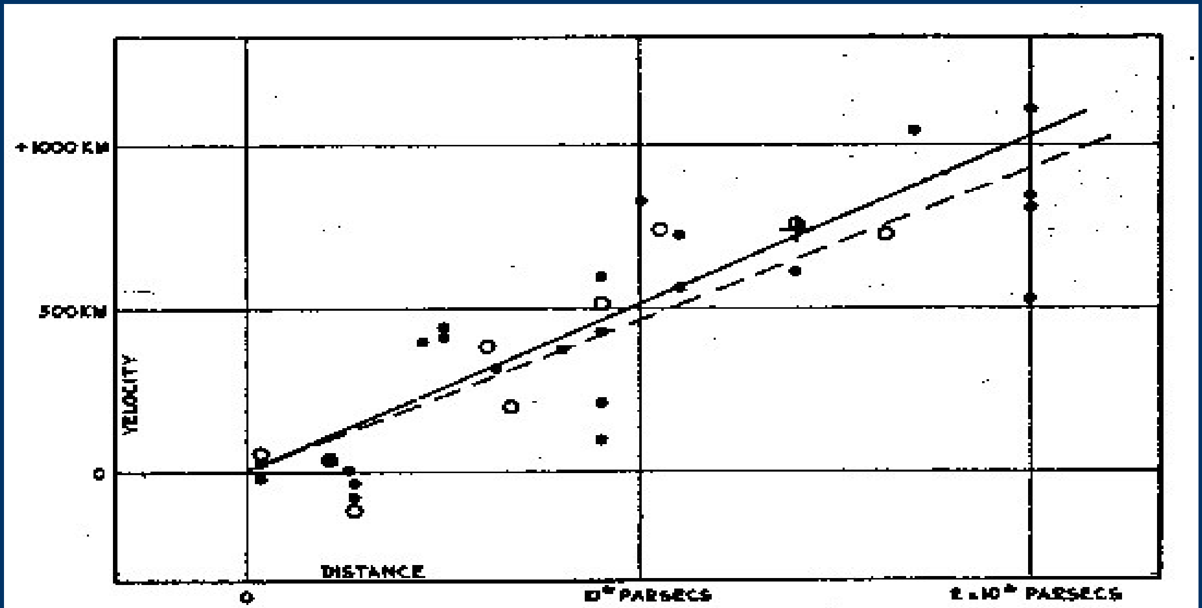


FIGURE 1

"The velocity-distance relation may represent the de Sitter effect. (...)
The necessary investigations are now under way in the odds,
for the moment, favoring de Sitter"

Summary of velocity-distance relations

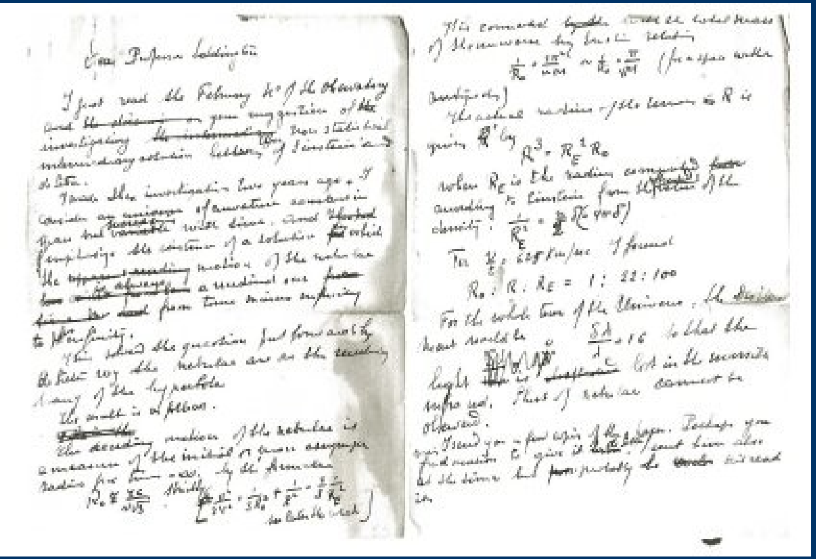
	velocity relation
Wirtz, 1924	$v = a - b \cdot \log(Dm)$
Lundmark, 1925	$v = k + lr + mr^2$
Strömberg, 1925	no definite relation
Hubble, 1929	$v = Kr$

4 – 1930: the expanding universe

- From Hubble's result: systematic recession
- 1930: Eddington and de Sitter proposed that the actual universe should be represented by an intermediate state between solution A and B
- Lemaître had already discovered in 1927 such a "third way":

Einstein universe (i.e. finite universe)
with a curvature radius
increasing with time and
evolving towards de Sitter universe

$R \equiv R(t)$



Letter from Lemaître to Eddington (1930)
[Courtesy: Louvain-la-Neuve Archives]

Friedmann-Lemaître equations
1927 Lemaître – 1922 Friedmann
(Einstein in 1923 and Robertson in 1929
rejected Friedmann solution)

$$3\frac{R'^2}{R^2} + \frac{3}{R^2} = \lambda + \kappa\rho$$

$$2\frac{R''}{R} + \frac{R'^2}{R^2} + \frac{1}{R^2} = \lambda - \kappa p$$

Expansion (Cosmological)
Redshift

rayon de l'univers. *Il est égal à l'excès sur l'unité du rapport des rayons de l'univers à l'instant où la lumière est reçue et à l'instant où elle est émise.* v est la vitesse de l'observateur qui produirait le même effet. Lorsque la source est suffisamment proche nous pouvons écrire approximativement

$$\frac{v}{c} = \frac{R_2 - R_1}{R_1} = \frac{dR}{R} = \frac{R'}{R}\,dt = \frac{R'}{R}\,r$$

où r est la distance de la source. Nous avons donc

$$\frac{R'}{R} = \frac{v}{cr} \qquad (23)$$

As from 1930:
decline of the interest in
de Sitter effect

Lemaître 1927

5 - Conclusion

- In the present historical study we reconstructed the debates which took place during the 1920's about relativistic cosmology, focusing in particular on the so-called “de Sitter effect” and the interpretation of redshift
- Although, after 1930, such an effect was seen by scientists involved during those years as an effect of “minor importance”, it was just such a property which foreshadowed a non-static picture of the universe as a whole
- Predictions and confirmations of a suitable redshift-distance relation marked the passage from a static picture of the universe to the expanding universe
- The de Sitter effect can be viewed as the leading and linking thread in the first comparison between observational cosmology and a suitable theory of the universe as a whole given by solutions of relativistic field equations
- In this perpective, cosmology evolved passing from speculation to empirical science during the 1920's, when the relativistic theoretical models of the universe were compared to astronomical measurements, and when some fundamental topics, which are still present in cosmology, were first faced in the debates about the de Sitter effect

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