Simulations of relativistic plasmas

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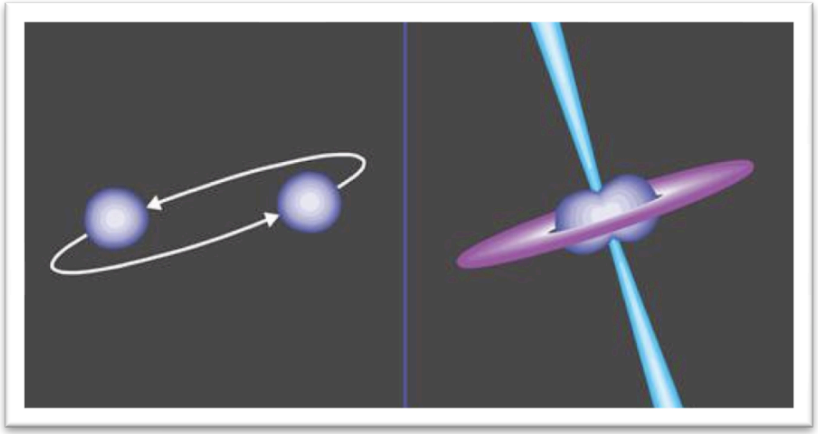
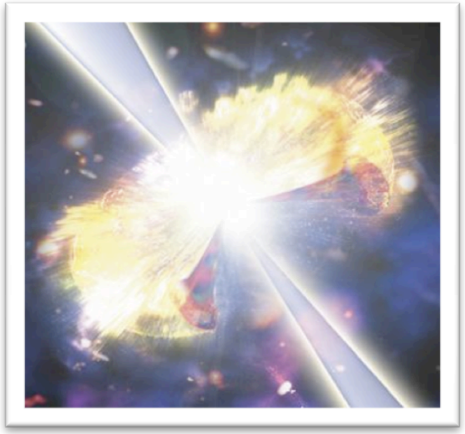
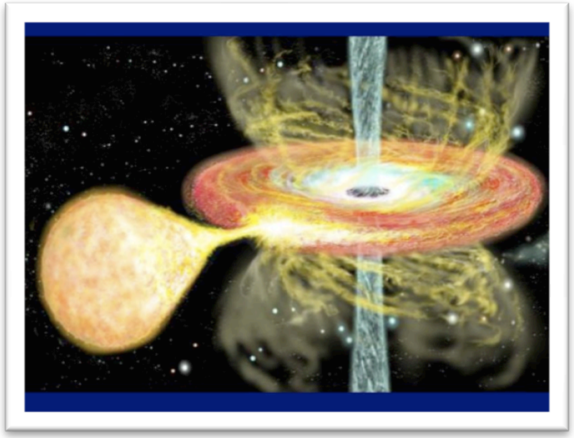
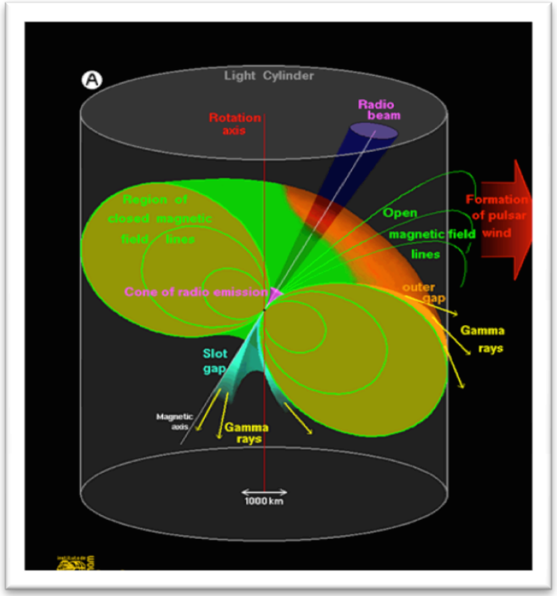
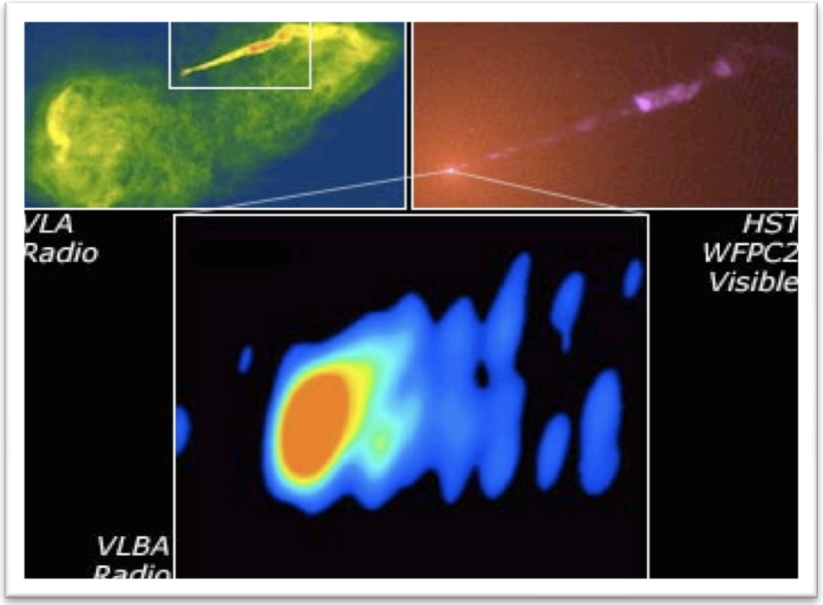
Compact objects and relativistic plasmas

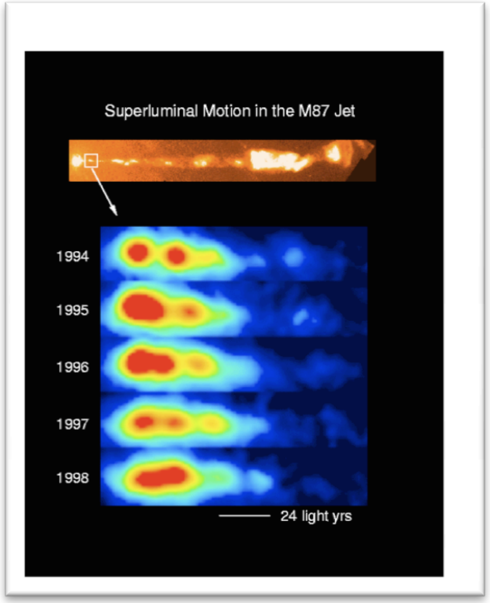












Fluid approximation: GRMHD

- GRMHD is a single-fluid ideal closure for plasmas. It is the relativistic extension of MHD, accounting in addition for:
  - Strong gravity, fast speeds, extremely hot plasmas, strong magnetic fields, electric force and displacement current
- Rapid growth in the last decade of numerical methods and codes for (E)GRMHD, applied to a variety of astrophysical situations involving compact objects:
  - Launching and propagation of AGN jets
  - Non-spherical accretion onto Black Holes
  - Jets from collapsars as GRB progenitors
  - Collapse to a Black Hole
  - Mergers of binary neutron stars

The Italian numerical community

- Simulations of relativistic magnetized plasmas:
  - Torino (University and Observatory): PLUTO code for classic and (special) relativistic MHD, simulations of AGN jets, development of numerical techniques
  - Firenze (University and Observatory): ECHO code for classic, relativistic and general relativistic MHD, simulations of Pulsar Wind Nebulae, proto-magnetar winds and GRB jets (with Berkeley), development of numerical techniques (with Bologna Observatory)

- General relativistic hydro simulations, GW emission:
  - Parma (University): neutron star formation, instabilities
  - Trieste (SISSA): proto-NS collapse, quark stars

Torino: AGN jets with the PLUTO code

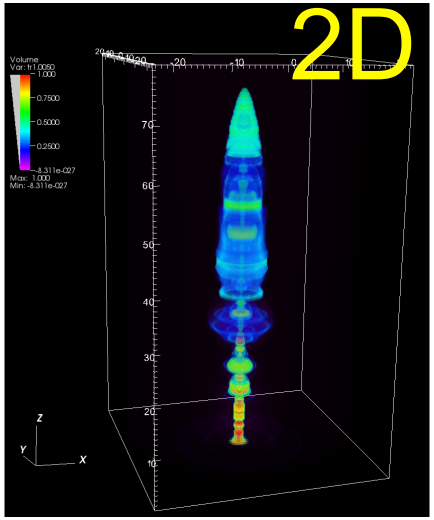
- ❑ Astrophysical target: FRI-FRII source dicothomy: wish to investigate braking of collimated outflows by external medium entrainment favoured by shear instabilities;
- ❑ Understanding the processes leading to momentum, energy and mass transfer to the environment is crucial and still largely unanswered → connecting morphology with deceleration;
- ❑ Using the PLUTO code (http://plutocode.oato.inaf.it) to investigate the propagation of relativistic magnetized supersonic jets using high resolution (640x1600x640) numerical simulations;
- ❑ Consider either purely toroidal or poloidal magnetic field configurations.

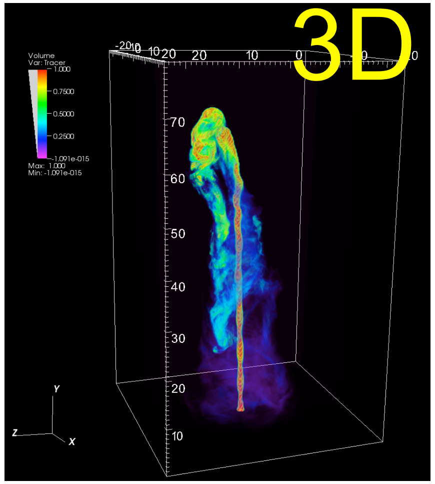
3D RMHD jet simulations

- Jet deceleration more efficient with increasing jet/ambient density contrast
- The presence of a poloidal (longitudinal) magnetic field does not affect considerably the evolution → similar to purely hydro case

- **<u>Toroidal field models</u>**:

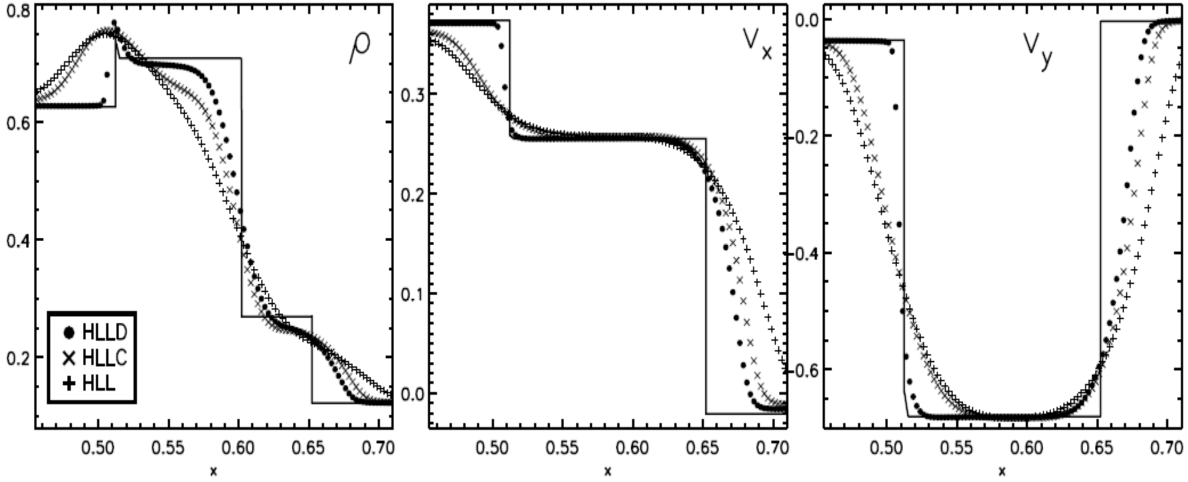
- typical 2D “nose cone” structures are not seen in 3D
- inhibits entrainment via shearing instabilities
- promotes strong backflow
- Jet “wiggling” (or deflection) via kink instability
- Able to re-accelerate the beam
- → Resolution plays a key role in these simulations!





Numerical methods: HLLD

- Development of a new 5 wave Riemann solver (HLLD) for RMHD.
- The solution to the Riemann problem is approximated by a five wave pattern, comprised of two outermost fast shocks, two rotational discontinuities and a contact surface in the middle.
- Proper closure to the Rankine-Hugoniot jump conditions can be attained by solving a nonlinear scalar equation in the total pressure variable.
- The new HLLD solver considerably improves over the popular HLL solver or the recently proposed HLLC schemes → Better resolution of Alfven waves.



Shock Tube problem: scheme comparison (1° order)

Firenze: The ECHO code

- Eulerian Conservative High Order code: the aim is to combine shock-capturing properties and accuracy for small scale wave propagation and turbulence, in a 3+1 approach
  - *L. Del Zanna, O. Zanotti, N. Bucciantini, P. Londrillo, 2007, A&A 473, 11*
  - GR upgrade of: *Londrillo & Del Zanna 2000* (MHD); *Del Zanna et al. 2003* (RMHD)
- Modular structure, F90 language, MPI parallelization
- Many physical modules (MHD, RMHD, GRMHD, GRMD,…)
- Any metric allowed (1-,2- or 3-D), even time-dependent
- Finite-difference scheme, Runge-Kutta time-stepping
- UCT strategy for the magnetic field (staggered grid)
- High-order reconstruction procedures (explicit and implicit)
- Central-type Riemann solvers (our most successful recipe!)

ECHO: Eulerian 3+1 approach for GRMHD

- Set of 8 conservation laws + 1 constraint:

$$\partial_t(\sqrt{\gamma}D)+\partial_i[\sqrt{\gamma}(\alpha v^i-\beta^i)D]=0$$

$$\partial_t(\sqrt{\gamma}S_j)+\partial_i[\sqrt{\gamma}(\alpha W^i_j-\beta^i S_j)]=\sqrt{\gamma}(\alpha W^{ik}\partial_j\gamma_{ik}/2+S_i\partial_j\beta^i-U\partial_j\alpha)$$

$$\partial_t(\sqrt{\gamma}U)+\partial_i[\sqrt{\gamma}(\alpha S^i-\beta^i U)]=\sqrt{\gamma}(\alpha K_{ij}W^{ij}-S^i\partial_i\alpha)$$

$$\partial_t(\sqrt{\gamma}B^j)+\partial_i[\sqrt{\gamma}(\alpha v^i-\beta^i)B^j-\sqrt{\gamma}(\alpha v^j-\beta^j)B^i]=0;\quad \partial_i(\sqrt{\gamma}B^i)=0$$

- No Lie derivatives nor Christoffel symbols needed in source terms
- The lapse function α, shift vector $\boldsymbol{\beta}$, metric tensor γ and the extrinsic curvature **K** may be time-dependent (evolved through Einstein's eqs.)
- Only familiar spatial 3-D vectors and tensors, easy RMHD and MHD limits

$$D=\rho\Gamma;\quad \vec{S}=\rho h\Gamma^2\vec{v}+\vec{E}\times\vec{B};\quad U=\rho h\Gamma^2-p+(E^2+B^2)/2$$

$$\overleftrightarrow{W}=\rho h\Gamma^2\vec{v}\vec{v}+p\vec{\gamma}-\vec{E}\vec{E}-\vec{B}\vec{B}+(E^2+B^2)/2\vec{\gamma};\quad \vec{E}=-\vec{v}\times\vec{B}$$

ECHO: discretization strategy

- The two sets of conservation laws are discretized in space according the Upwind Constrained Transport strategies (UCT: *Londrillo & Del Zanna ApJ 530, 508, 2000; JCP 195, 17, 2004*)
  - Staggered grid for magnetic and electric field components
  - Finite differences: point values at cell centers (u), at cell faces (b and f), at edges (e). The *hat* indicates high-order differencing

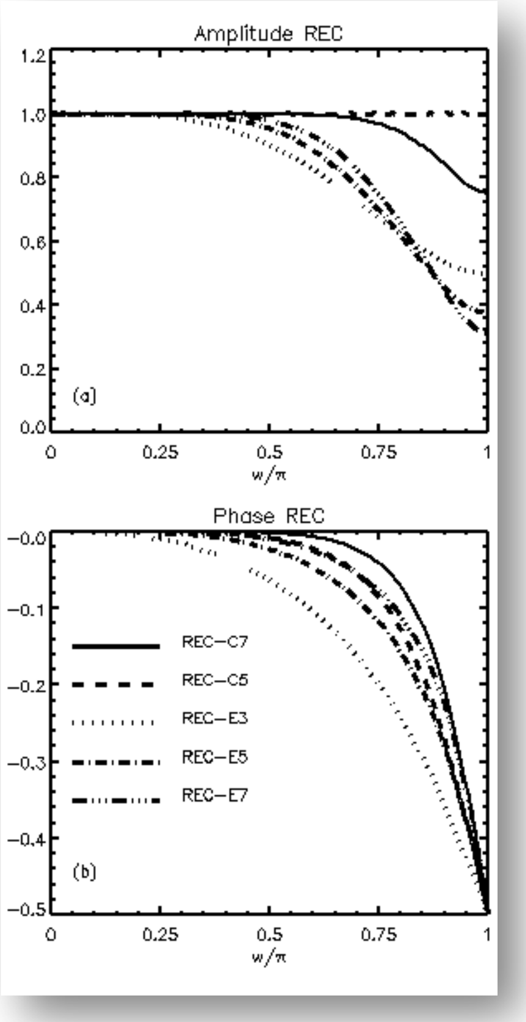
$$\boxed{\frac{d}{dt}[u_j]_C + \sum_i \frac{1}{h_i}\left([\hat{f}_j^i]_{S_i^+} - [\hat{f}_j^i]_{S_i^-}\right) = [s_j]_C}$$

$$\boxed{\frac{d}{dt}[b^i]_{S_i^+} + \sum_{j,k}[ijk]\frac{1}{h_j}\left([\hat{e}_k]_{L_k^+} - [\hat{e}_k]_{L_k^-}\right) = 0; \quad \sum_i \frac{1}{h_i}\left([\hat{b}^i]_{S_i^+} - [\hat{b}^i]_{S_i^-}\right) = 0}$$

  - The solenoidal constraint is maintained algebraically at any order
  - A four-state numerical flux is required for electric field components

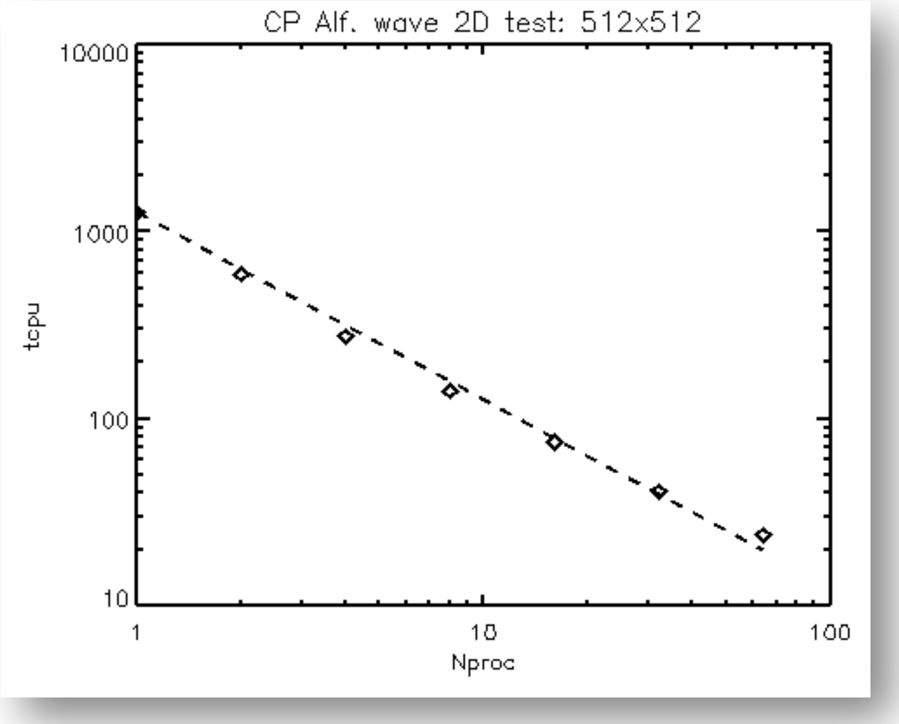
ECHO: high-order procedures

- Three 1-D component-wise procedures:
    - RECconstruction (upwind L-R primitive vars)
    - DERivation (centered, needed for *hat* fluxes)
    - INTerpolation (centered, needed from b to B)
- Library of REC routines implemented:
    - TVD-like (MinMod, Monotonized Centered)
    - WENO/CENO (*Jiang Shu 1996, Liu Osher 1998*)
    - Fixed explicit stencils (r=3,5,7) + MP filter (Monotonicity Preserving: *Suresh Huyn 1997*)
    - Compact implicit routines (*Lele 1992*) with spectral-like resolution + MP filter
- No system-dependent characteristics!



ECHO: parallelization

- Compact routines ⇒ dimensional swapping (all to all)
- MPI library calls (just a couple), compiled only for parallel runs
- Test on IBM SP5 (CINECA): 512x512 simulation up to 64 PEs



$$\boxed{t_{cpu} \propto N^{-1}}$$

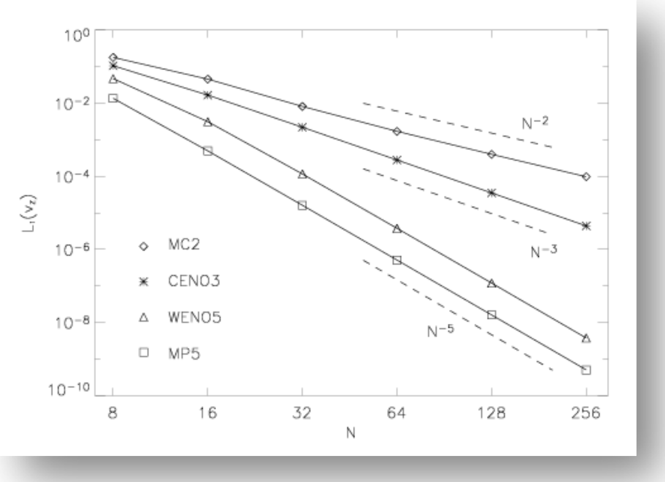
Numerical tests: convergence

- A large-amplitude CP Alfvén wave is an exact solution for both MHD and RMHD (here E is important, Va is modified)
- Convergence is measured on any quantity u after one period T of propagation along the diagonal of a 2-D periodical box:

$$\boxed{L_1(u) = \textstyle\sum_{ij} | u(x_i, y_j, T) - u(x_i, y_j, 0) | / N^2 \propto N^{-r}}$$

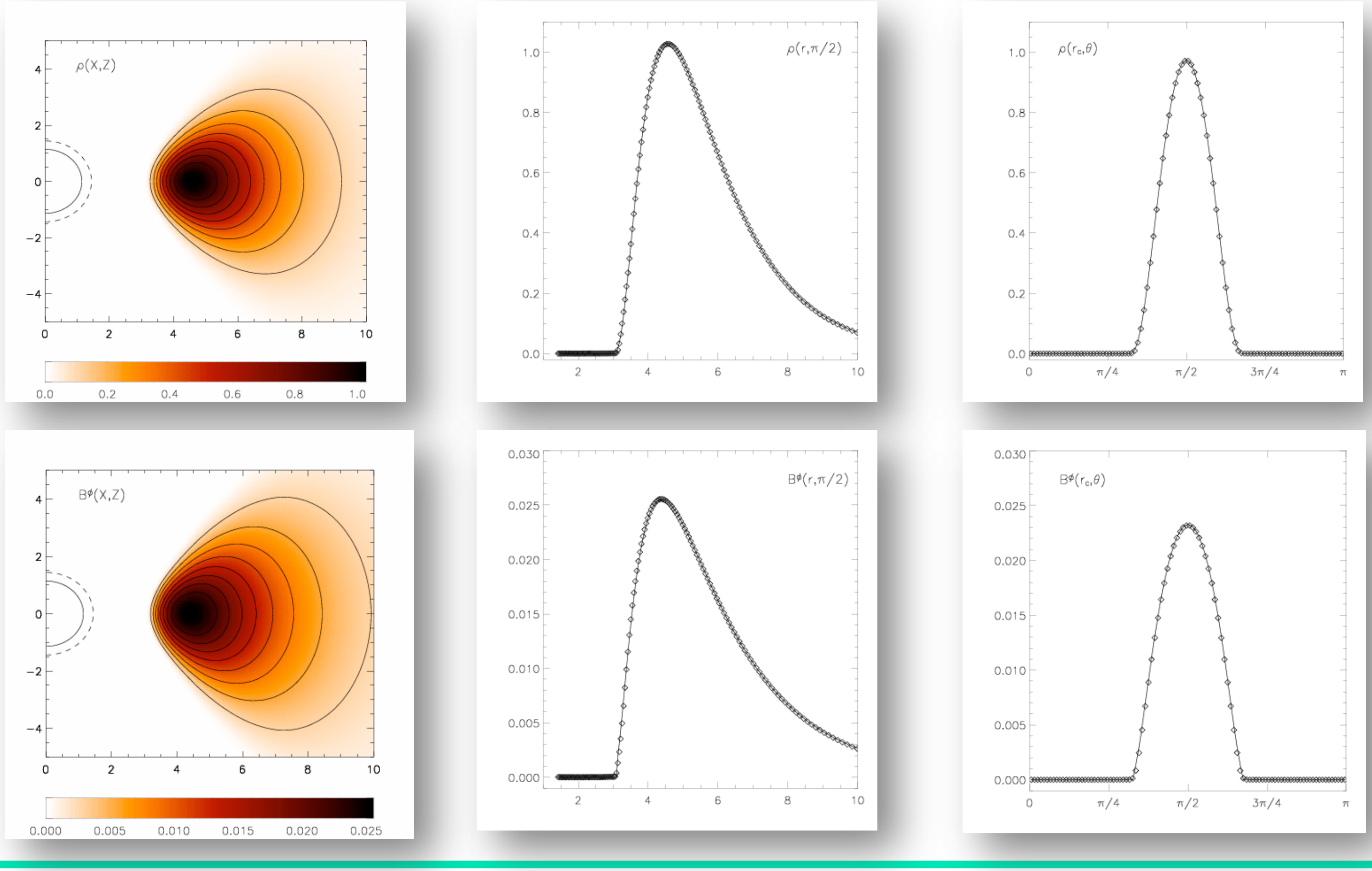
$$B_y = \eta B_0 \cos(x - v_A t), \quad v_y = -v_A B_y / B_0$$
$$B_z = \eta B_0 \sin(x - v_A t), \quad v_z = -v_A B_z / B_0$$

$$\boxed{v_A^2 = \frac{B_0^2}{\rho h + (1+\eta^2) B_0^2} \left\{ \frac{1 + \sqrt{1 - 4\eta^2 B_0^2 / [\rho h + (1+\eta^2) B_0^2]}}{2} \right\}^{-1}}$$

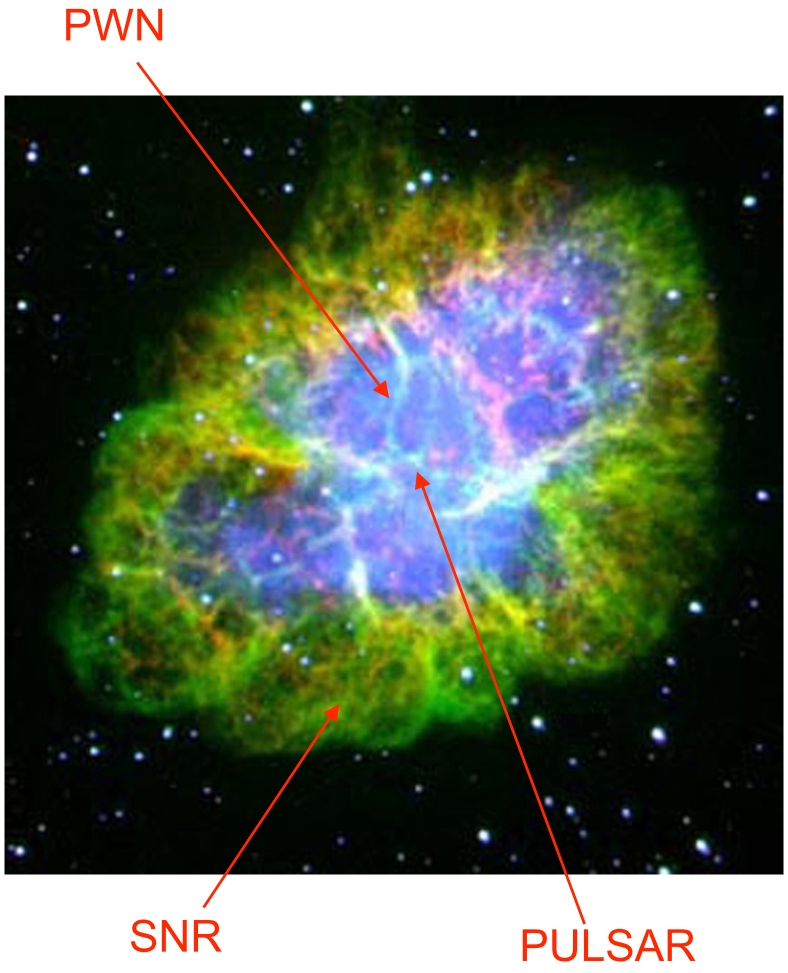


Numerical tests: thick disk in Kerr metric

- Results for t=200, approximately 3 rotation periods, with MP5, RK2



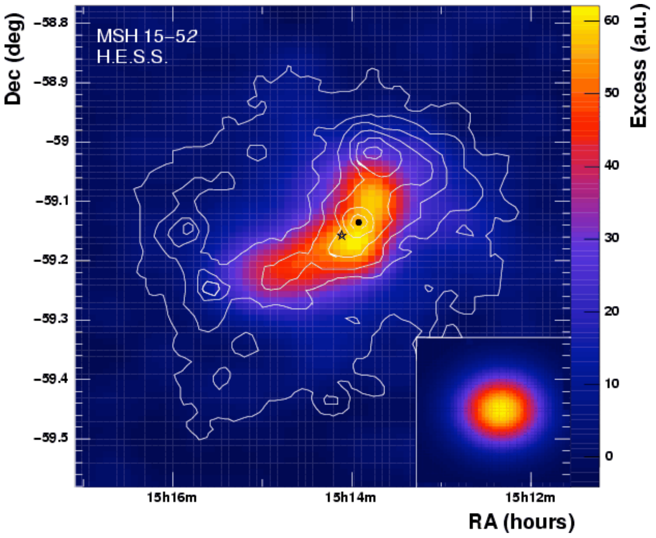
RMHD model of Pulsar Wind Nebulae



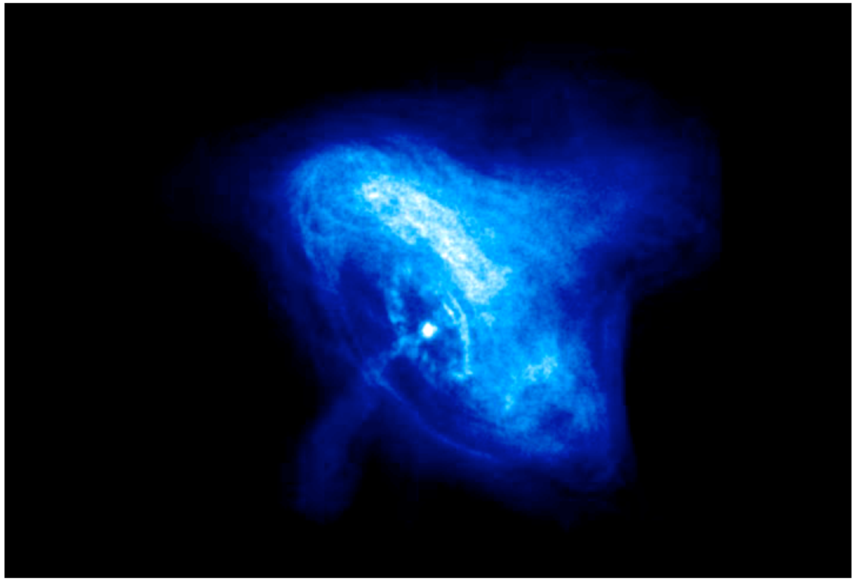
- PWNe are hot bubbles (also called plerions) of relativistic particles and magnetized plasma emitting non-thermal radiation (synchrotron - IC) from radio to γ
- Originated by the interaction of the ultra-relativistic magnetized pulsar wind with the expanding SNR (or with the ISM)
- Crab Nebula in optical: central amorphous mass (continuum) and external filaments (lines)

γ-ray observations

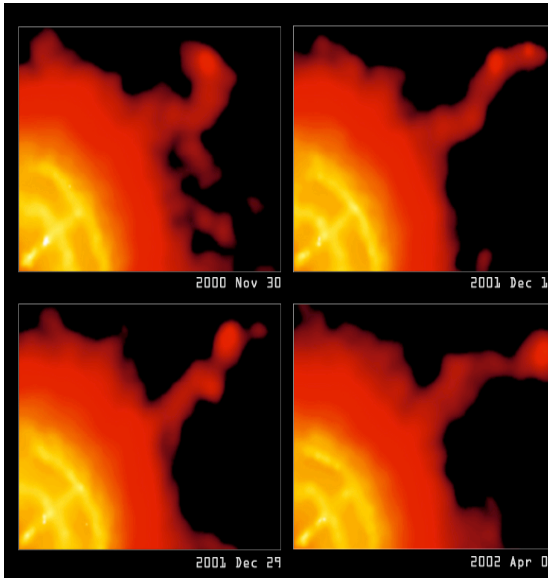
- PWNe are also sources of γ-ray emission (e.g. HESS)
  - MeV photons produced by high-energy tail of synchrotron
  - GeV-TeV photons produced by IC scattering of background light
  - Possible hadronic component (ions in pulsar wind?)
- Particles (pairs) with up to ~10^10 MeV energies required!
  - Physics of particle acceleration
  - Infos on distribution function
  - Independent diagnostics on B field



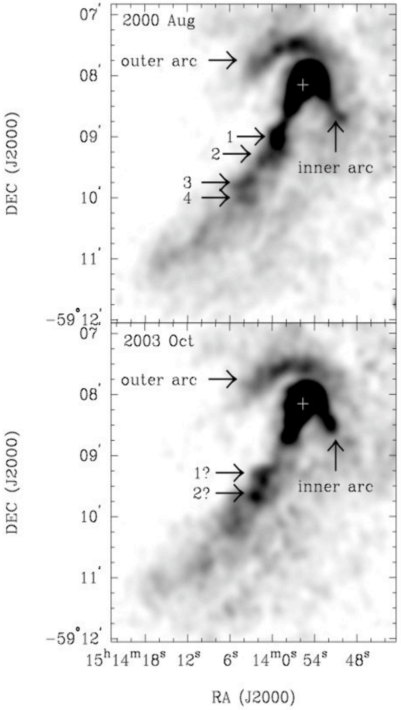
Jet-torus structure and relativistic motions



Crab



Vela



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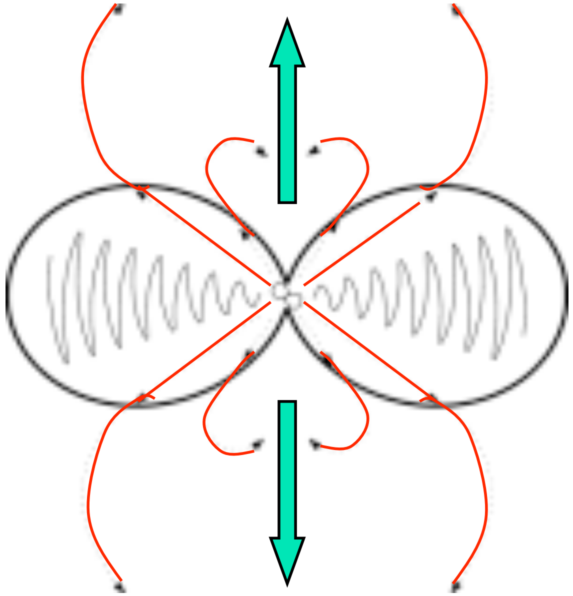
- Chandra (X): axisymmetric jet-torus structure!
- Equatorial motions (wisps): v=0.3-0.5 c, 0.5-0.8c in jets
  - Timescale of months-year: MHD or gyrating ions?

Jet-torus structure: theory



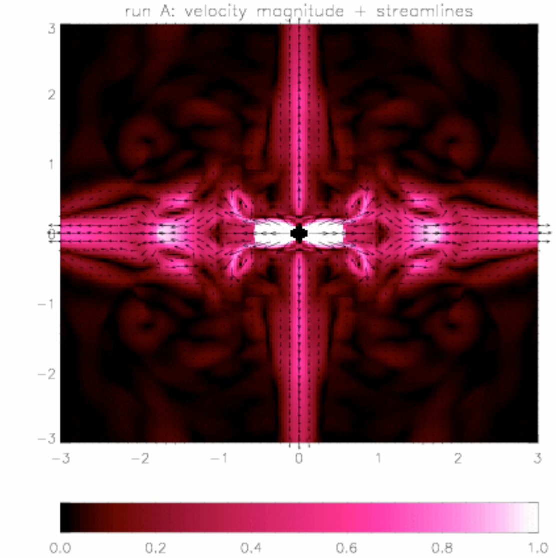
- Torus: higher equatorial energy flux
- Jets: magnetic collimation. But in PW:

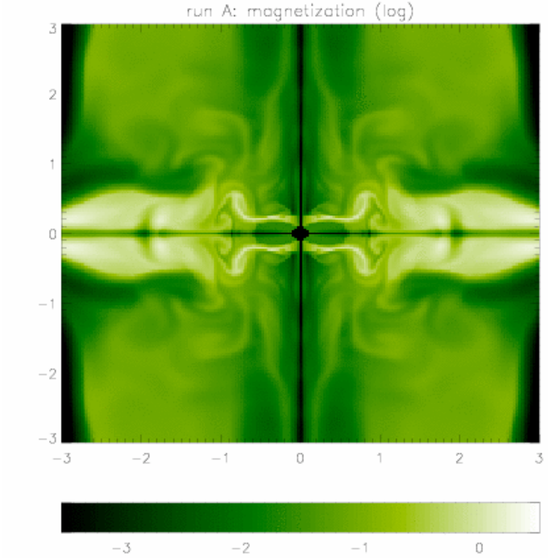
$$\gamma >> 1 \Rightarrow \rho_q \vec{E} + \vec{j} \times \vec{B} \approx 0$$

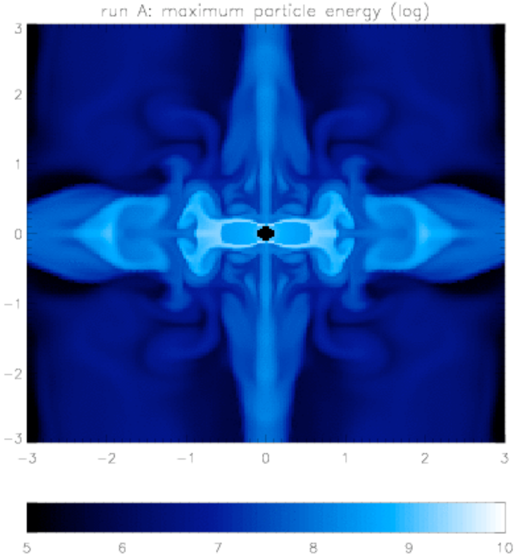
collimation downstream of the TS?
  - *Lyubarsky, 2002*
  - *Bogovalov & Khangoulian, 2002, 2003*
- Axisymmetric RMHD simulations of the interaction of an anisotropic relativistic magnetized wind with SN ejecta
  - *Komissarov & Lyubarsky, 2003, 2004*
  - *Del Zanna et al. 2004*

Simulated dynamics and energy losses

- Particles are injected at TS with $\varepsilon_\infty = E / m_e c^2 = 10^{10}$
- Stronger synchrotron losses occur along TS and in the torus, where magnetization is higher
- The flow pattern allows emission also in polar jets

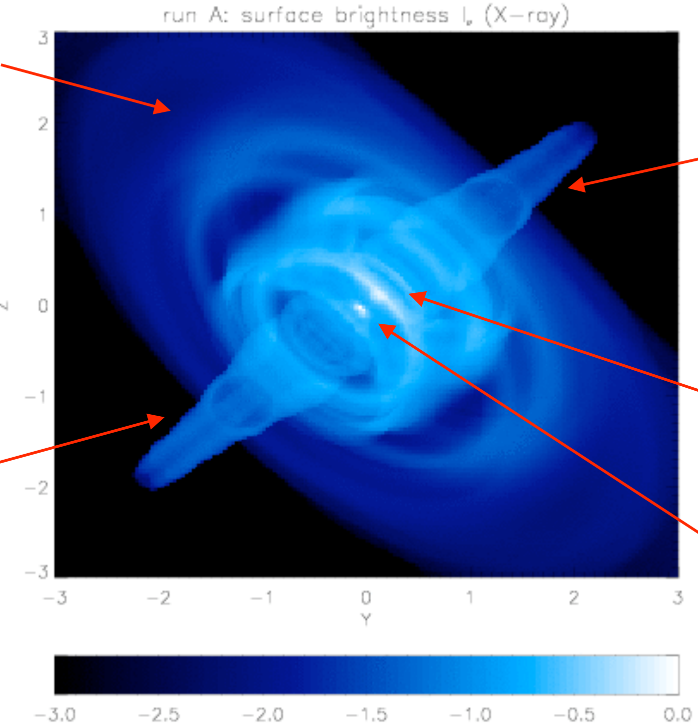

velocity


magnetization


max energy

Comparison with observations (Crab)

- Jet-torus structure reproduced in synchrotron X-ray maps
- Diagnostics: synchrotron and IC non-thermal emission
  - Del Zanna et al. 2006; Volpi et al. 2008



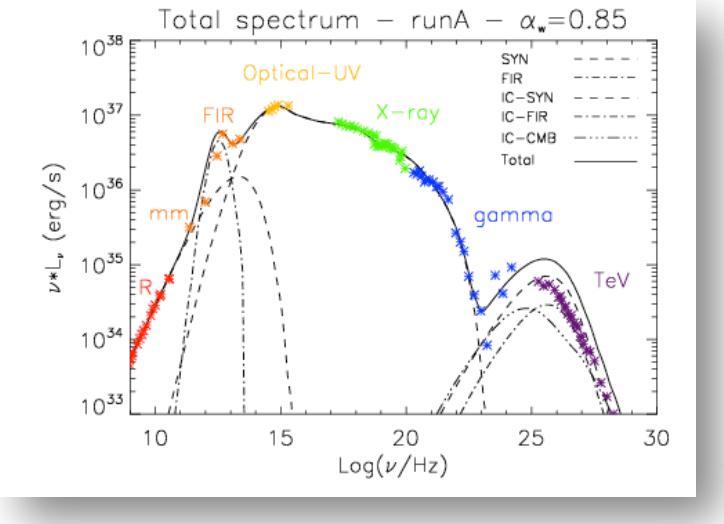
torus

counter-jet

jet

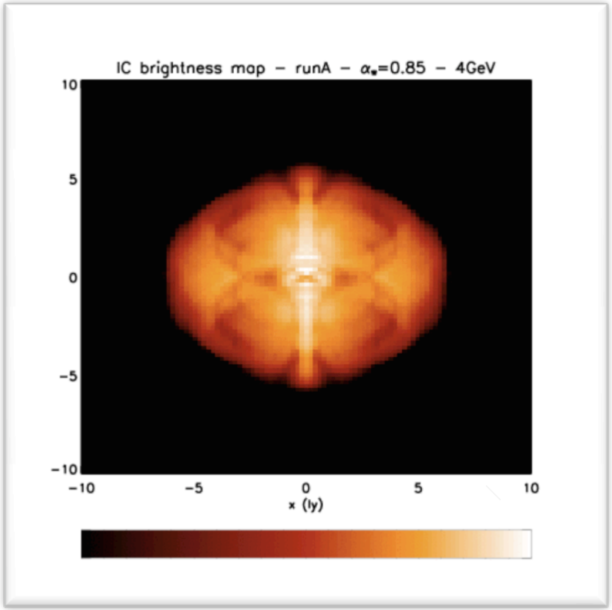
Inner ring

knot

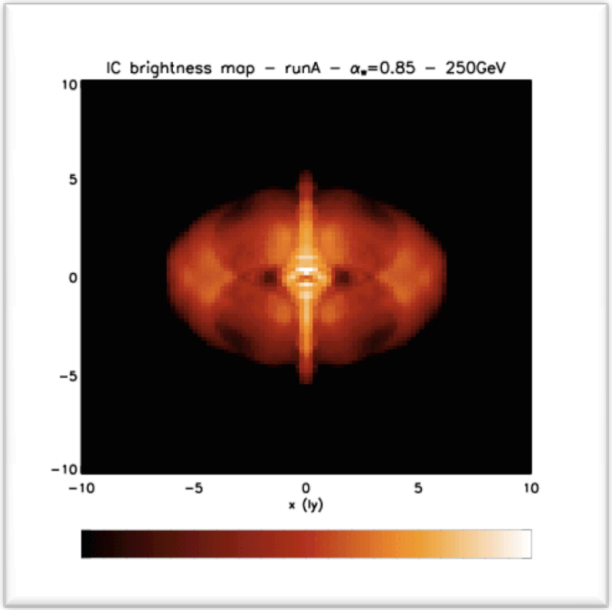


The overall Crab spectrum

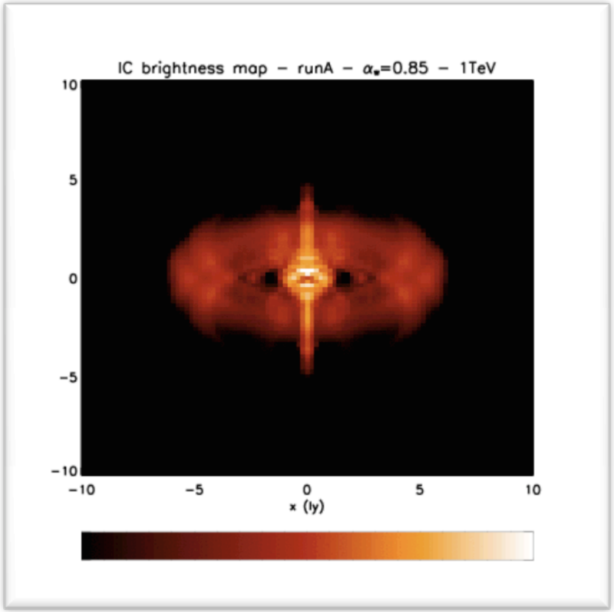
First γ-ray surface brightness maps



4 GeV



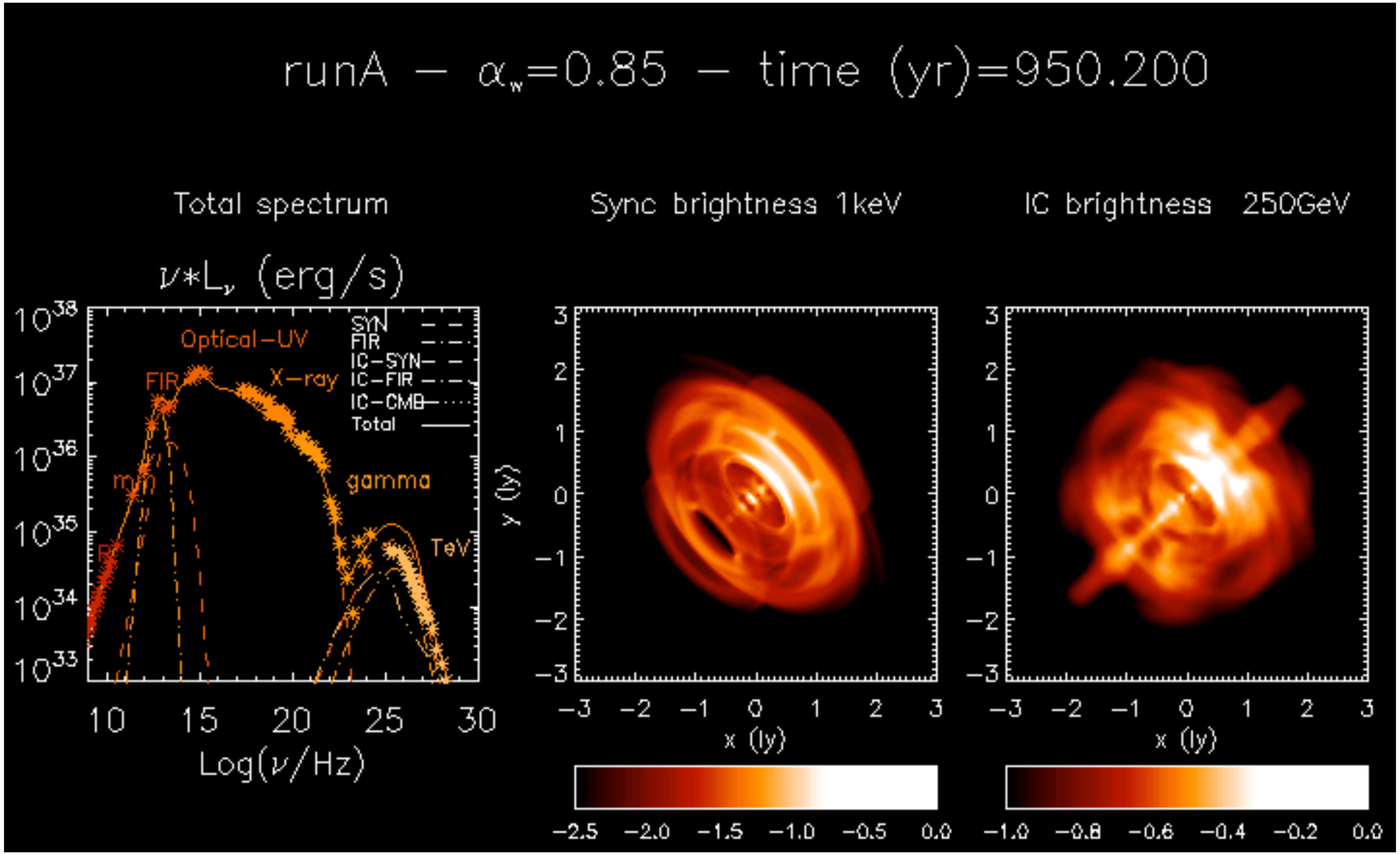
250 GeV



1 TeV

- Jet-torus structure predicted in γ-rays
- Shrinkage of PWN size with increasing frequency

Time variability: MHD origin



LGRB – SN connection: what drives the jet?

Accretion onto a BH:
Collapsar model

Accretion power can provide the correct energy
Accretion disks are associated with jets
Accretion can be sustained for a long time

Spinning Neutron Star:
Magnetar model

Millisecond rotating magnetars can provide the correct energy (spin-down)
Pulsar wind are highly relativistic
Spin-down can last for a long time
NSs are born during core-collapse SNe

The main question is: What is the fate of massive progenitors? BH or NS?

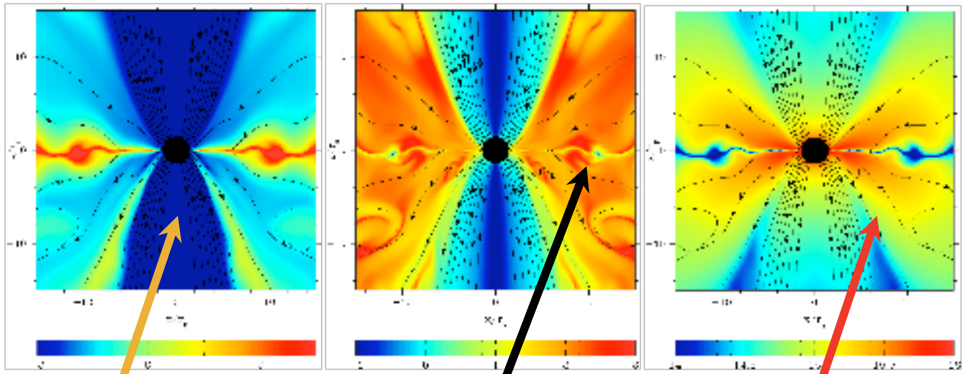
The difference depend on the mass of the progenitor

Standard models predict that progenitor with masses > 10 Msun should form a BH, however these models neglect the role and efficiency of mass loss

Other Important questions regard the driving mechanism:

Neutrino-Antineutrino annihilation vs MHD magneto-centrifugal acceleration

The collapsar scenario



Komissarov & Barkov 2007

Evacuated jet

Accreting torus

Disk wind

GRB energy comes from BH rotation

Jet streamlines originate from BH
Blandford-Znajek powers the jet
Collimation is due to the torus-wind

Pro:

Collapse in high mass stars favors BH
Jets naturally associated with accretion disks
Very high Γ can be achieved in the jet
Fragmentation of the torus can lead to late time accretion events (flares)
Accretion can be sustained for a long time

Cons:

Need rapidly rotating BH
Γ is set by non obvious mass loading
Need ordered seed magnetic field
Need a long surviving torus inside SN
Direct collapse to BH does not obviously produce the SN shock

Proto-magnetars and GRB jets

- Long duration GRBs could be generated by proto-magnetar winds collimating polar relativistic jets which finally escape from the stellar progenitor
- Same magnetic pinching effect due to toroidal fields as in PWNe
- Axisymmetric RMHD simulations with assigned wind conditions (*Bucciantini et al. 2008*):
  $$\dot{E} = 10^{51}\ erg/s, \quad \Gamma_w = 10, \quad \sigma = 0.1$$
- Full wind + jet evolution available (*Komissarov & Barkov 2008, Bucciantini et al. 2009*)

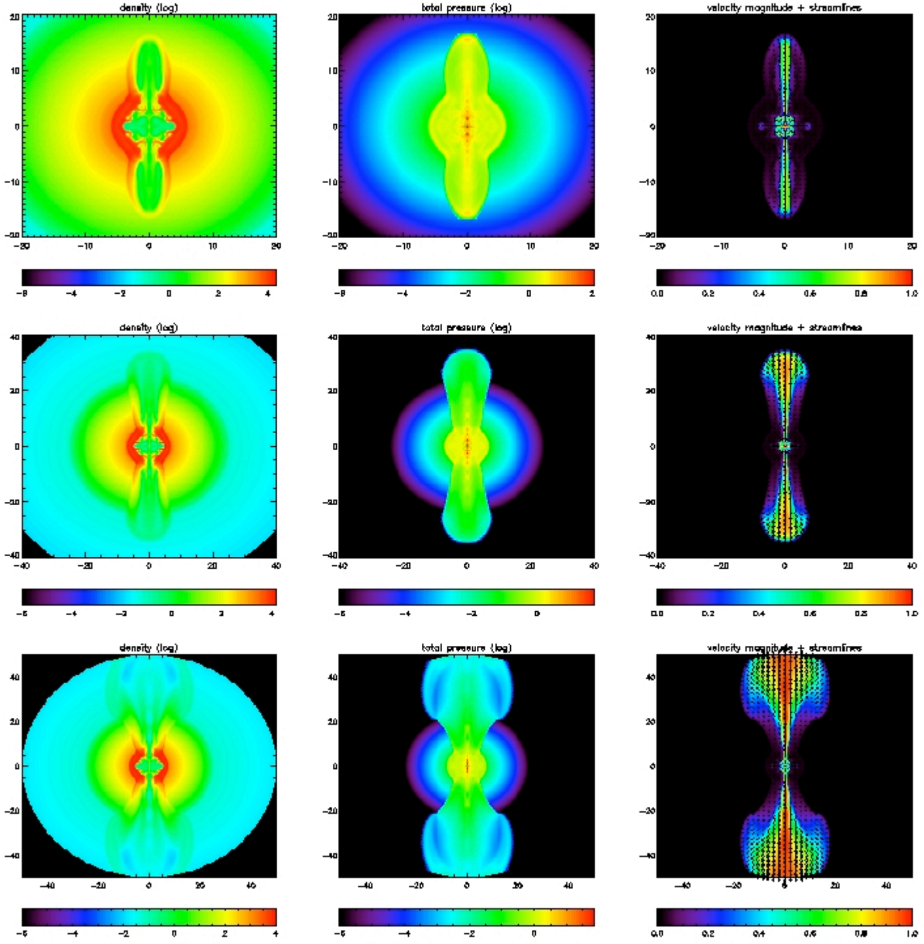
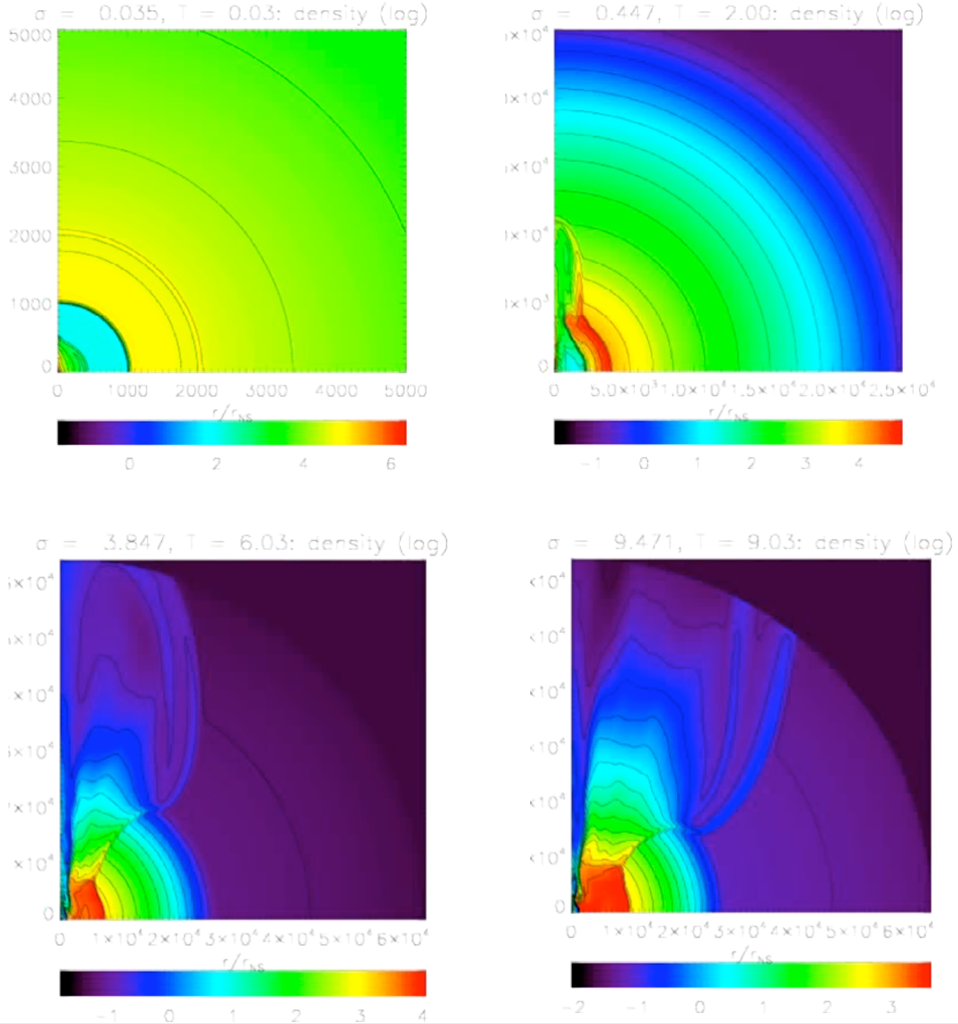


Figure 1. Evolution of a magnetized bubble inflated by a magnetar wind with $\dot{E} = 10^{51}$ erg s^{-1}, $\gamma_w = 10$ and $\sigma = 0.1$, inside a 35-M$_\odot$ progenitor star. From left to right: density (g cm^{-3}), pressure (g cm^{-3} c^2) and velocity (in units of c). From top to bottom: snapshots at 4, 5 and 6 s after core bounce. Distances are in 10^9 cm; the radius of the progenitor star is 2.5×10^{10} cm. By $t = 5$ s (middle panel) the jet has escaped the progenitor star.

Proto-magnetars and GRB jets: full case

- First GRMHD simulation of a magnetar wind and GRB jet: from the NS surface up to the stellar atmosphere (almost a factor 10^5 in radius!)
- Engine: from a thermally driven (neutrino heating simulated via an isothermal hot layer) to a centrifugally driven MHD wind
- Pinching beyond TS in MWN
- Simulation from t=1s after core bounce up to t=10s, when the relativistic jet has finally left the stellar progenitor



Conclusions

- Relativistic plasmas ubiquitous in High-Energy Astrophysics: sources of non-thermal emission and particle acceleration
- Two Italian groups are leading experts in numerical modeling of magnetized relativistic plasmas: Torino and Firenze
- Torino: PLUTO code, AGN jets
- Firenze: ECHO code, PWNe and LGRB jet engines
- Future: code merging? Tough…
- Future: coupling to Einstein solvers?

Thank you!